

## Gravitational quasinormal modes of AdS black branes in $d$ spacetime dimensions

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

JHEP09(2009)117

(<http://iopscience.iop.org/1126-6708/2009/09/117>)

[The Table of Contents](#) and [more related content](#) is available

Download details:

IP Address: 80.92.225.132

The article was downloaded on 01/04/2010 at 13:41

Please note that [terms and conditions apply](#).

# Gravitational quasinormal modes of AdS black branes in $d$ spacetime dimensions

Jaqueline Morgan,<sup>a</sup> Vitor Cardoso,<sup>b,c</sup> Alex S. Miranda,<sup>d</sup> C. Molina<sup>e</sup> and  
Vilson T. Zanchin<sup>a</sup>

<sup>a</sup>Centro de Ciências Naturais e Humanas, Universidade Federal do ABC,  
Rua Santa Adélia 166, 09210-170 Santo André, SP, Brazil

<sup>b</sup>Centro Multidisciplinar de Astrofísica - CENTRA, Dept. de Física,  
Instituto Superior Técnico,  
Avenida Rovisco Pais 1, 1049-001 Lisboa, Portugal

<sup>c</sup>Department of Physics and Astronomy, The University of Mississippi, University,  
MS 38677-1848, U.S.A.

<sup>d</sup>Instituto de Física, Universidade Federal do Rio de Janeiro,  
Caixa Postal 68528, RJ 21941-972, Brazil

<sup>e</sup>Escola de Artes, Ciências e Humanidades, Universidade de São Paulo,  
Avenida Arlindo Bettio 1000, 03828-000 São Paulo, SP, Brazil

E-mail: [jaqueline.morgan@ufabc.edu.br](mailto:jaqueline.morgan@ufabc.edu.br), [vcardoso@fisica.ist.utl.pt](mailto:vcardoso@fisica.ist.utl.pt),  
[astmiranda@if.ufrj.br](mailto:astmiranda@if.ufrj.br), [cmolina@usp.br](mailto:cmolina@usp.br), [zanchin@ufabc.edu.br](mailto:zanchin@ufabc.edu.br)

**ABSTRACT:** The AdS/CFT duality has established a mapping between quantities in the bulk AdS black-hole physics and observables in a boundary finite-temperature field theory. Such a relationship appears to be valid for an arbitrary number of spacetime dimensions, extrapolating the original formulations of Maldacena's correspondence. In the same sense properties like the hydrodynamic behavior of AdS black-hole fluctuations have been proved to be universal. We investigate in this work the complete quasinormal spectra of gravitational perturbations of  $d$ -dimensional plane-symmetric AdS black holes (black branes). Holographically the frequencies of the quasinormal modes correspond to the poles of two-point correlation functions of the field-theory stress-energy tensor. The important issue of the correct boundary condition to be imposed on the gauge-invariant perturbation fields at the AdS boundary is studied and elucidated in a fully  $d$ -dimensional context. We obtain the dispersion relations of the first few modes in the low-, intermediate- and high-wavenumber regimes. The sound-wave (shear-mode) behavior of scalar (vector)-type low-frequency quasinormal mode is analytically and numerically confirmed. These results are found employing both a power series method and a direct numerical integration scheme.

**KEYWORDS:** Classical Theories of Gravity, Black Holes, p-branes, AdS-CFT Correspondence

---

## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>The background spacetime</b>	<b>3</b>
<b>3</b>	<b>Fundamental equations for the gravitational perturbations</b>	<b>4</b>
3.1	Metric perturbations	5
3.1.1	Tensorial sector	5
3.1.2	Vectorial sector	5
3.1.3	Scalar sector	5
3.2	Master equations for the RWZ variables	6
3.2.1	Tensorial sector	6
3.2.2	Vectorial sector	7
3.2.3	Scalar sector	7
3.3	Master equations for the KS variables	7
3.3.1	Tensorial sector	8
3.3.2	Vectorial sector	8
3.3.3	Scalar sector	8
<b>4</b>	<b>Gauge-invariant variables and boundary conditions</b>	<b>9</b>
4.1	Tensorial sector	10
4.2	Vectorial sector	10
4.3	Scalar sector	11
4.3.1	Four dimensions	11
4.3.2	Five dimensions	12
4.3.3	Six and higher dimensions	12
<b>5</b>	<b>The quasinormal spectra: analytical results</b>	<b>12</b>
5.1	The hydrodynamic limit	13
5.1.1	Tensorial perturbations	13
5.1.2	Vectorial perturbations	13
5.1.3	Scalar perturbations	14
5.2	Asymptotic analysis of the QNM	14
5.2.1	Small wavenumbers, large frequencies	14
5.2.2	Large number of spacetime dimensions	15
<b>6</b>	<b>Numerical results</b>	<b>15</b>
6.1	Methods	15
6.1.1	Power series method	15
6.1.2	Time evolution method	16
6.2	Numerical results for the tensorial QNM	17

6.3	Numerical results for the vectorial QNM	19
6.4	Numerical results for the scalar QNM	22
6.5	More comments on the numerical results	25
<b>7</b>	<b>QNM and the AdS/CFT correspondence</b>	<b>26</b>
7.1	Thermalization timescale	26
7.1.1	Vectorial sector	27
7.1.2	Scalar sector	27
7.1.3	Tensorial sector	28
7.2	Causality in the dual CFT plasma	28
<b>8</b>	<b>Final comments and conclusion</b>	<b>29</b>

---

## 1 Introduction

The anti-de Sitter/conformal field theory (AdS/CFT) correspondence [1–3] has been widely recognized as an important tool to explore a variety of non-perturbative aspects of strongly coupled gauge theories. Holographic string-theory models are now used to study the physics of strong interactions and condensed matter, both in the zero- and finite-temperature regimes (see, e.g., refs. [4–10] for reviews and lecture notes on AdS/CFT applications to QCD and condensed matter physics). One of the essential ingredients of this approach is the weak/strong relationship between the coupling constants [11]. When the 't Hooft coupling of the large- $N$  CFT is strong, string theory on AdS spacetime reduces to classical supergravity theory. In such context one can investigate diverse phenomena in a class of strongly interacting field theories by doing the computations on the gravity side of the correspondence. Among other results, this procedure has allowed the determination of near-equilibrium properties of the dual CFT plasma, such as transport coefficients like viscosity, conductivity and diffusion constants [12–17].

The AdS/CFT correspondence is also used to study fundamental questions in gravitational physics, which are hard or even impossible to be addressed within current gravity theories, such as the nature of spacetime singularities [18–22] and the loss of information in black holes [23, 24]. Even in regimes for which it is possible to obtain results from gravity theories, the AdS/CFT correspondence leads to new interpretations of those results. One example is the evolution of classical fields in the neighborhood of asymptotically AdS black holes (black branes). The vectorial sector of gravitational perturbations presents a fundamental quasinormal mode (QNM) frequency which is purely damped and goes to zero in the small wavenumber limit. The unusual behavior of this mode was not understood from a strictly gravitational point of view [25–28]. However, based on the AdS/CFT duality and taking into account the expected features of low-energy fluctuations in interacting field theories, Policastro, Son and Starinets [13, 14] were able to interpret such quasinormal mode as the dual of the shear transverse mode predicted by relativistic fluid mechanics.

Some works [29, 30] then suggested that the old ‘membrane-paradigm’ framework [31–33], in which the (stretched) horizon is interpreted as a fluid, could be used to explain the hydrodynamic properties of black holes, and this very important concept was incorporated in the physics of dynamical classical fields in AdS spacetimes.

The hydrodynamic behavior of AdS black hole fluctuations, in particular the universality of such a behavior for four, five, and seven spacetime dimensions, both at first- and second-order expansions in the frequency and momentum [34–39], is now a generally accepted property of AdS black holes (see also refs. [40–47] for recent developments in the fluid/gravity correspondence). However, there are several other features of the quasinormal spectra of AdS black holes and black branes which were not considered in higher dimensional spacetimes, especially in the case of gravitational perturbations. Such a study is important because, among others, it allows one to verify if there are aspects of the spectra which are specific to a given spacetime dimension, or which aspects are dimension-independent. One example is the crossover from the hydrodynamic regime to a “collisionless regime” appearing in four and five spacetime dimensions [48–51]. For any event-horizon size (or temperature), there is a critical wavenumber value above which the late-time evolution of the vector-type gravitational fluctuations is dominated by the first gapped quasinormal mode and not by the hydrodynamic shear mode. A possible extension of this crossover for perturbations of higher-dimensional AdS black holes (and black branes) has not been investigated yet. This is one of the goals of the present work.

There are other important issues in the study of the vibrational modes of AdS black branes to be analyzed in a fully  $d$ -dimensional context. We can mention the arbitrariness in the choice of gauge-invariant combinations of metric variations as fundamental variables of the gravitational perturbations. Another related issue is the ambiguity in defining an appropriate condition for the quasinormal modes at AdS spacetime boundary. Historically, the perturbation variables are chosen in such a way that the radial part of the fundamental equations takes a Schrödinger-like form when written in terms of the Regge-Wheeler tortoise coordinate. These are called the Regge-Wheeler-Zerilli (RWZ) variables. In some of the works on this subject [52–55] the authors have chosen RWZ type variables and argued that, according to the AdS/CFT duality, the conditions to be imposed at AdS boundary are such that gravitational perturbations do not deform the boundary metric. In the four-dimensional case, Michalogiorgakis and Pufu [53] showed that a Robin boundary condition is the correct condition to be imposed on the RWZ master variable governing the scalar-type perturbations. With such a boundary condition they were able to obtain, for instance, the hydrodynamic wave sound mode which had not been obtained in early works using RWZ variables and a Dirichlet condition at AdS boundary [25–28]. A different route was taken in refs. [51, 56], where the ambiguities characteristic of classical-field dynamics at AdS spacetimes were eliminated by defining the quasinormal (QN) frequencies as the poles, in the space of frequency and momentum, of retarded Green functions in the dual field theory. In this approach, the standard tools to compute real-time Green functions from holography [57–59] are used in order to find the correct boundary conditions that should be imposed on metric perturbations at the AdS boundary. Any set of perturbation functions chosen to fulfill these requirements are called Kovtun-Starinets (KS) variables.

In particular, it was shown that Dirichlet boundary conditions and KS type variables lead to the correct quasinormal spectra of AdS black branes in four and five spacetime dimensions [51, 56]. These and other related subjects are investigated here considering AdS black branes in spacetimes of arbitrary number of dimensions.

The present work also aims to address other issues. For instance:

- (i) establish (numerically) the stability of black branes against scalar-type perturbations, a result that was proved only for four-dimensional black holes and black branes [60];
- (ii) analyze the causality of signal propagation in the dual CFT plasma using recent results on the eikonal limit of the QNM spectra [61, 62];
- (iii) use a time evolution method to investigate the absence/presence of power-law tails at late stages of the evolution of perturbations in higher dimensional AdS black branes, and;
- (iv) search for the highly real modes found analytically by Daghighi and Green [63, 64], but were not confirmed numerically until this moment.

The structure of this work is the following. In the next section we define the  $d$ -dimensional AdS black brane spacetime and the conventions adopted in the main body of the work. In section 3 it is presented the one-dimensional Schrödinger-like equations obtained in ref. [65] for the RWZ master variables. The same section is also devoted to obtain the fundamental equations for the Kovtun-Starinets variables using the partially covariant and totally gauge-invariant formalism of Kodama, Ishibashi and Seto [66]. In section 4 we analyze which boundary conditions should be imposing on KS and RWZ variables in order to obtain the same spectrum in each perturbation sector. The analysis is performed for an arbitrary number of spacetime dimensions. Section 5 is devoted to report a few interesting analytical results. The numerical results are presented and analyzed in section 6. In section 7 the QNM are analyzed in terms of the AdS/CFT correspondence, and in the section 8 we make final comments and conclude.

## 2 The background spacetime

The background spacetime considered here represents a  $d$ -dimensional plane-symmetric asymptotically anti-de Sitter (AdS) black hole, or simply an AdS black brane [67–72]. The spacetime can be locally written as a product of a two-dimensional spacetime  $\mathcal{N}^2$ , spanned by a timelike coordinate  $t$  and a radial spacelike coordinate  $r$ , and a  $(d - 2)$ -dimensional space  $\mathcal{K}^{d-2}$  with constant sectional curvature  $K = 0$  [65, 66]. With such a decomposition, the background metric in Schwarzschild-like coordinates takes the form

$$ds^2 = \frac{r^2}{R^2} \left[ -f(r) dt^2 + \sum_{i=2}^{d-1} dx^i dx_i \right] + \frac{R^2}{r^2 f(r)} dr^2, \tag{2.1}$$

for which

$$f(r) = 1 - \frac{r_h^{d-1}}{r^{d-1}}, \tag{2.2}$$

with  $r_h$  being the event horizon radius, and  $R$  the AdS radius. The coordinates  $x^i$ ,  $i = 2, 3, \dots, d-1$ , span the  $\mathcal{K}^{d-2}$  space.

The Hawking temperature of the black brane is

$$T = \frac{(d-1)r_h}{4\pi R^2}, \tag{2.3}$$

and the AdS radius  $R$  is given in terms of the negative cosmological constant  $\Lambda$  through the relation

$$R^2 = -\frac{(d-2)(d-1)}{2\Lambda}. \tag{2.4}$$

The radial coordinate  $r$  covers, without singularities, the whole region of interest for the analysis of the QNM of the AdS black hole of metric (2.1), namely, the range  $(r_h, \infty)$ . To simplify the analysis, as usual we introduce a new coordinate which is defined in a finite interval. This is done through the following re-parameterization

$$u = \frac{r_h}{r}, \tag{2.5}$$

which results in

$$f(u) = 1 - u^{d-1}. \tag{2.6}$$

Now the event horizon is located at  $u = 1$ , and the AdS spatial infinity ( $r \rightarrow \infty$ ) is at  $u = 0$ . Hence, we have  $u \in (1, 0)$ , and the metric (2.1) becomes

$$ds^2 = \frac{r_h^2}{u^2 R^2} \left[ -f(u) dt^2 + \sum_{i=2}^{d-1} dx^i dx_i \right] + \frac{R^2}{u^2 f(u)} du^2. \tag{2.7}$$

In the following, coordinates  $(t, u)$  are labeled as  $x^a$ ,  $a = 0, 1$ , i.e., coordinates  $t$  and  $u$  span the relevant region of  $\mathcal{N}^2$  outside the horizon.

The foregoing black-brane spacetime has been extensively studied in the last years in connection with the AdS/CFT correspondence, specially for  $d = 4, 5$  and  $7$  dimensions. In such a cases the metric (2.7) can be seen as part of nonextremal solutions to the supergravity equations of motion in ten or eleven dimensions [11, 73]. The near-horizon limit of the full supergravity spacetime is the direct product of an  $\text{AdS}_d$  black brane and a  $S^{D-d}$  sphere, where  $D = 10$  for  $d = 5$  and  $D = 11$  for  $d = 4$  and  $7$ . The internal degrees of freedom corresponding to the  $(D-d)$ -dimensional sphere will not be important for the present work, since we are interested in the correlators of the CFT energy-momentum tensor and, according to the gauge/gravity dictionary [5, 74], this operator is dual to the gravitational fluctuations of the background spacetime (2.7).

The general properties of metric perturbations of the considered  $d$ -dimensional black branes are investigated in the next section where we write the fundamental equations that govern the evolution of gravitational perturbations in these asymptotically AdS spacetimes.

### 3 Fundamental equations for the gravitational perturbations

Following the procedure presented in ref. [66], the gravitational perturbations are expanded in terms of harmonic functions on  $\mathcal{K}^{d-2}$  and the first-order perturbed Einstein equations are

given in terms of a set of gauge-invariant quantities. These quantities are combinations of the metric perturbations  $h_{\alpha\beta}$  which are related to the perturbed spacetime metric through the usual definition  $g_{\alpha\beta} = g_{\alpha\beta}^{(0)} + h_{\alpha\beta}$ , where  $g_{\alpha\beta}^{(0)}$  stands for the background metric defined by eq. (2.7). The gravitational perturbations are grouped into three distinct classes (sectors) according to the special type of harmonic tensors that appear in the expansions of  $h_{\alpha\beta}$ . These can be tensorial, vectorial, or scalar perturbations, corresponding respectively to the scalar, shear and sound symmetry channels for the gravitational fluctuations considered in ref. [56]. Each one of these sectors is governed by a particular closed group of independent differential equations. It is possible to choose a particular set of master variables which allows to write only one perturbation equation for each perturbation sector, as, for instance, the RWZ set of variables adopted in ref. [65]. Another interesting set of variables (KS variables) were first used in ref. [56]. Here we present the fundamental equations for KS variables in  $d$  spacetime dimensions, and establish a connection between RWZ and KS variables. The gauge-invariant metric perturbation nomenclature and labelling follows ref. [66].

### 3.1 Metric perturbations

#### 3.1.1 Tensorial sector

This particular set of gravitational perturbations can be represented in terms of tensorial harmonics  $\mathbb{T}_{ij}$ , in the form

$$h_{ab} = 0, \quad h_{ai} = 0, \quad h_{ij} = 2u^{-2}H_T\mathbb{T}_{ij}, \quad (3.1)$$

where  $H_T = H_T(t, u)$  is a gauge-invariant function depending on the coordinates  $t$  and  $u$  only, and  $\mathbb{T}_{ij}$  are transverse traceless harmonic tensors defined on  $\mathcal{K}^{d-2}$  [66].

#### 3.1.2 Vectorial sector

Metric perturbations of vectorial type can be expanded in terms of vectorial harmonic functions  $\mathbb{V}_i$  as follows

$$h_{ab} = 0, \quad h_{ai} = u^{-1}f_a\mathbb{V}_i, \quad h_{ij} = 2u^{-2}H_V\mathbb{V}_{ij}, \quad (3.2)$$

where  $f_a = f_a(t, u)$  and  $H_V = H_V(t, u)$  are scalar functions of the coordinates on  $\mathcal{N}^2$ , to be determined, and  $\mathbb{V}_{ij}$  are vector-type harmonic tensors on  $\mathcal{K}^{d-2}$  built from the transverse harmonic vectors  $\mathbb{V}_i$  (see ref. [66]). From the functions  $f_a$  and  $H_V$  it is defined a new set of gauge-invariant quantities  $F_a$  ( $a = 0, 1$ ) given by

$$F_a = f_a + \frac{1}{uk}D_aH_V, \quad (3.3)$$

where  $k$  is the perturbation wavenumber, and  $D_a$  is the covariant derivative in the space  $\mathcal{N}^2$ .

#### 3.1.3 Scalar sector

Scalar gravitational perturbations are the set of metric perturbations which can be expanded in terms of scalar harmonic functions  $\mathbb{S}$  in the form

$$h_{ab} = f_{ab}\mathbb{S}, \quad h_{ai} = u^{-1}f_a\mathbb{S}_i, \quad h_{ij} = 2u^{-2}(H_L\gamma_{ij}\mathbb{S} + H_S\mathbb{S}_{ij}), \quad (3.4)$$



where  $f_{ab} = f_{ab}(t, u)$ ,  $f_a = f_a(t, u)$ ,  $H_L = H_L(t, u)$  and  $H_S = H_S(t, u)$  are functions to be determined.  $\mathbb{S}_i$  and  $\mathbb{S}_{ij}$  are respectively scalar-type harmonic vectors and tensors on  $\mathcal{K}^{d-2}$  built from the scalar harmonic functions  $\mathbb{S}$  (see ref. [66]). A set of gauge-invariant quantities are then defined as

$$F = H_L + \frac{1}{n}H_S + uD^a \left( \frac{1}{u} \right) X_a, \tag{3.5}$$

$$F_{ab} = f_{ab} + D_a X_b + D_b X_a,$$

with

$$X_a = \frac{1}{uk} \left( f_a + \frac{1}{uk} D_a H_S \right). \tag{3.6}$$

Now we write the gravitational fundamental equations for each perturbation sector and each set of variables, RWZ and KS.

### 3.2 Master equations for the RWZ variables

Kodama and Ishibashi [65] showed that for a black brane in four or more spacetime dimensions, the Einstein equations for the gravitational perturbations can be reduced to three independent second-order wave equations in a two-dimensional static spacetime, one equation corresponding to each one of the perturbation modes. Moreover, the variable for the final second-order master equation for a specific mode is given by a simple combination of gauge-invariant quantities in the formalism of ref. [66]. It is introduced new variables  $\Phi_p$  so that, after Fourier decomposition of such perturbation functions,  $\Phi_p(t, u) = \int \Phi_p(u) e^{i\omega t} d\omega$ , the perturbation equations take a Schrödinger-like form,

$$\frac{d^2 \Phi_p}{d\tau_*^2} + (\mathfrak{w}^2 - V_p) \Phi_p = 0, \tag{3.7}$$

where  $\tau_*$  is the normalized tortoise radial coordinate, defined by  $du/d\tau_* = -f(u)$ . The label  $p$  can be  $T$ ,  $V$  or  $S$  depending of the perturbation sector: tensorial, vectorial and scalar, respectively.  $V_p$  is the effective potential, and the parameter  $\mathfrak{w}$  is the normalized frequency defined by

$$\mathfrak{w} = \frac{(d-1)\omega}{4\pi T} = \frac{R^2}{r_h} \omega, \tag{3.8}$$

where  $T$  stands for the Hawking temperature of the black brane.

Next we define the RWZ variable  $\Phi_p$  for each perturbation sector and the corresponding effective potentials.

#### 3.2.1 Tensorial sector

As argued in ref. [65], the simplest function  $\Phi_T$  that allows to write the resulting perturbation equation in a Schrödinger-like form is

$$\Phi_T = u^{-\frac{d-2}{2}} H_T, \tag{3.9}$$

where  $H_T$  (introduced in eqs. (3.1)) is a gauge-invariant quantity by itself [66]. In such a case, the potential  $V_T$  (cf. eq. (3.7)) for this sector is given by

$$V_T(u) = f(u) \left[ \mathfrak{q}^2 + \frac{d(d-2)}{4u^2} + \frac{(d-2)^2 u^{d-3}}{4} \right]. \quad (3.10)$$

Here the parameter  $\mathfrak{q}$  is the normalized wavenumber defined by

$$\mathfrak{q} = \frac{(d-1)k}{4\pi T} = \frac{R^2}{r_h} k. \quad (3.11)$$

### 3.2.2 Vectorial sector

The variable  $\Phi_V$  is defined implicitly by (see ref. [65])

$$F^a = u^{d-3} \epsilon^{ab} D_b \left( u^{-\frac{d-2}{2}} \Phi_V \right), \quad (3.12)$$

where  $F^a$  is the gauge-invariant quantity defined in eq. (3.3) [66], and  $\epsilon^{ab}$  is the Levi-Civita tensor in the two-space  $\mathcal{N}^2$ . The corresponding effective potential for the vectorial sector  $V_V$  is

$$V_V(u) = f(u) \left[ \mathfrak{q}^2 + \frac{(d-2)(d-4)}{4u^2} - \frac{3(d-2)^2 u^{d-3}}{4} \right]. \quad (3.13)$$

### 3.2.3 Scalar sector

In this sector, the RWZ variable  $\Phi_S$  suggested in ref. [65] is given by

$$\Phi_S = \frac{2(d-2) u^{-\frac{d-4}{2}}}{2\mathfrak{q}^2 + (d-1)(d-2)u^{d-3}} \left( \frac{2F}{u} + \frac{if(u)F_{ut}}{\mathfrak{w}} \right), \quad (3.14)$$

where  $F$  and  $F_{ut}$  are the gauge-invariant quantities given by eqs. (3.5) [66], and the effective potential is

$$V_S(u) = \frac{f(u)Q(u)}{4[2\mathfrak{q}^2 + (d-1)(d-2)u^{d-3}]^2}. \quad (3.15)$$

Here  $Q(u)$  is given by

$$Q(u) = (d-2)^3 [d + (d-2)(1-f)] \frac{f'^2}{u^4} - 4(d-2) [(d-5)(d-2)(d-1) + (d-2)^2 f - 4(1-f)] \frac{f'}{u^3} \mathfrak{q}^2 + 4(d-6) [d-4-3(d-2)(1-f)] \frac{1}{u^2} \mathfrak{q}^4 + 16\mathfrak{q}^6. \quad (3.16)$$

## 3.3 Master equations for the KS variables

Another choice of fundamental variables for the gravitational perturbations was firstly suggested by Kovtun and Starinets [56]. A set of master equations for the Kovtun-Starinets (KS) variables in  $d = 4$  and 5 dimensions were obtained in refs. [51, 56]. In connection with the formalism of ref. [66], we present here the fundamental equations for the KS variables in  $d$  spacetime dimensions.

### 3.3.1 Tensorial sector

For  $d$  spacetime dimensions the KS variable for the tensorial sector  $Z_T$  is defined by

$$Z_T = H_T/2, \quad (3.17)$$

where  $H_T$  is the gauge-invariant quantity introduced in eqs. (3.1). In terms of  $Z_T(u)$ , defined by  $Z_T(t, u) = \int Z_T(u) e^{i\omega t} d\omega$ , the perturbation equation for the tensorial sector is given by

$$Z_T'' - \left[ \frac{d-1-f}{uf} \right] Z_T' + \left[ \frac{\mathfrak{w}^2 - \mathfrak{q}^2 f}{f^2} \right] Z_T = 0, \quad (3.18)$$

where the primes indicate derivatives with respect to the coordinate  $u$ , and  $f = f(u)$  is the horizon function defined in eq. (2.6). This equation reduces to the corresponding perturbation equation of ref. [56] when one takes  $d = 5$  and makes the adjustments for different notation and normalizations.

### 3.3.2 Vectorial sector

In connection with the formalism developed by Kodama, Ishibashi and Seto [66], the KS master variable for the vectorial gravitational perturbations takes the form

$$Z_V = F_t/u, \quad (3.19)$$

where  $F_t$  is the gauge-invariant quantity of ref. [66], as defined in eq. (3.3). With this variable, after Fourier decomposition as in the tensorial case, we obtain the following equation

$$Z_V'' - \left[ \frac{d-2}{u} + \frac{f'\mathfrak{w}^2}{f(\mathfrak{q}^2 f - \mathfrak{w}^2)} \right] Z_V' + \left[ \frac{\mathfrak{w}^2 - \mathfrak{q}^2 f}{f^2} \right] Z_V = 0, \quad (3.20)$$

which is the master equation for the vectorial metric perturbations of  $d$ -dimensional black branes in terms of the KS gauge-invariant variable  $Z_V$ . The general expression eq. (3.20) reduces to the equations for vector perturbations in four and five spacetime dimensions, as seen in [51, 56].

### 3.3.3 Scalar sector

Again inspired in the work by Kovtun and Starinets [56], we write the following gauge-invariant quantity to describe the scalar-type gravitational perturbations,

$$Z_S = u^2 F_{tt} + \left[ (d-1)u^{d-1} + 2f(u) \right] F, \quad (3.21)$$

where  $F_{tt}$  and  $F$  are gauge-invariant quantities defined in eqs. (3.5) [66]. With this expression it is shown that  $Z_S$  satisfies the following differential equation

$$Z_S'' + \frac{Y_1 \mathfrak{q}^2 + Y_2 \mathfrak{w}^2}{ufX} Z_S' + \frac{Y_3 \mathfrak{q}^2 + Y_4 \mathfrak{q}^4 + 2(d-2)\mathfrak{w}^4}{f^2 X} Z_S = 0, \quad (3.22)$$

where we have introduced the coefficients

$$\begin{aligned}
 X &= 2(d-2)\mathfrak{w}^2 - [d-3 + (d-1)f]\mathfrak{q}^2, \\
 Y_1 &= 2(d-2)^2 f^2 + (d-1)(d-1+f)u^{d-1}, \\
 Y_2 &= -2(d-2)[d-1-f], \\
 Y_3 &= -(d-3)f'^2 f - [4(d-2)f + (d-1)u^{d-1}]\mathfrak{w}^2, \\
 Y_4 &= [2(d-2) + (d-1)u^{d-1}]f.
 \end{aligned}$$

It is important to mention that eq. (3.22) can be reduced to the scalar perturbation equation of [51] when one takes  $d = 4$ , and to the corresponding equation of [56] when one takes  $d = 5$ .

#### 4 Gauge-invariant variables and boundary conditions

It is known that ingoing wave condition at horizon and Dirichlet condition at AdS boundary applied to the Regge-Wheeler-Zerilli (RWZ) variables does not give all the QNM of a given perturbative sector due to the choice of the boundary condition at spatial infinity [51]. In particular, the sound wave mode in four dimensional spacetimes does not show up [25–28]. On the other hand, using the same boundary conditions and Kovtun-Starinets (KS) gauge-invariant variables the mentioned sound wave mode appeared [16, 51], and it was also verified that in some cases RWZ and KS yield different non-hydrodynamic quasinormal frequencies [51]. In these circumstances one must be able to decide which spectrum has a meaningful physical interpretation. In accordance to the AdS/CFT correspondence we opt for the QN frequencies obtained from the poles of the related two-point correlation functions, i.e., we choose the spectrum obtained by applying an ingoing wave condition at horizon and a Dirichlet boundary condition at  $u = 0$  to the KS gauge-invariant variables  $Z_p(u)$  [56]. However, it is known that different master variables can lead in special cases to the same spectrum, as it happens with the polar and axial gravitational perturbations of asymptotically flat four-dimensional black holes [75]. Having this in mind the objective in this section is to investigate which boundary conditions must be applied to each variable in order to produce the QNM spectrum corresponding to the poles of the stress-energy tensor correlators in the dual field theory. A comparison among the spectra obtained with the same boundary conditions applied to the RWZ quantities  $\Phi_p(u)$  and to the KS variables  $Z_p(u)$  is also done. The first step is then to find the asymptotic form of the perturbation functions  $Z_p$ , i.e., we try solutions of the form  $Z_p \sim u^\nu$ , where  $\nu$  is a parameter to be determined. We find that eqs. (3.18), (3.20) and (3.22) are satisfied in the limit  $u \rightarrow 0$  if  $\nu = 0$ , or if  $\nu = d - 1$ . Therefore the solutions for  $Z_p(u)$  which satisfy the incoming-wave condition at horizon, here denoted by  $Z_p^{in}(u)$ , present the following asymptotic behavior around  $u = 0$ :

$$Z_p^{in}(u) = \mathcal{A}_p(\mathfrak{w}, \mathfrak{q}) + \dots + \mathcal{B}_p(\mathfrak{w}, \mathfrak{q})u^{d-1} + \dots, \tag{4.1}$$

where  $p = T, V, S$  refers respectively to the tensorial, vectorial, and scalar perturbation sectors. The ellipses in the foregoing equation denote higher powers of  $u$ , and quantities

$\mathcal{A}_p(\mathfrak{w}, \mathfrak{q})$  and  $\mathcal{B}_p(\mathfrak{w}, \mathfrak{q})$  are the connection coefficients related to the respective differential equations. After eqs. (4.1) one finds that Dirichlet boundary conditions imposed on  $Z_p^{in}(u)$  give

$$Z_p^{in}(0) = \mathcal{A}_p(\mathfrak{w}, \mathfrak{q}) = 0, \quad p = T, V, S. \quad (4.2)$$

The next step is to study the relations among the KS  $Z_p(u)$  and the RWZ  $\Phi_p(u)$  variables at the AdS boundary and to find the relations between the QNM spectra, which we call the KS- and the RWZ-spectra, respectively, for short. We do that by considering separately each one of the perturbation sectors.

#### 4.1 Tensorial sector

For the tensorial gravitational sector, we were able to find an explicit relation between the RWZ variable  $\Phi_T(u)$  and the KS gauge-invariant variable  $Z_T(u)$  for any spacetime dimension. It is given by

$$Z_T(u) = \frac{1}{2} u^{\frac{d-2}{2}} \Phi_T(u). \quad (4.3)$$

Furthermore, it can be shown from eqs. (3.7) and (3.10) that at the asymptotic region  $\Phi_T^{in}(u)$  is of the form

$$\Phi_T^{in}(u) = \mathcal{C}_T(\mathfrak{w}, \mathfrak{q}) u^{-\frac{d-2}{2}} + \dots + \mathcal{D}_T(\mathfrak{w}, \mathfrak{q}) u^{\frac{d}{2}} + \dots, \quad (4.4)$$

agreeing with the asymptotic form for  $d$  spacetime dimensions found in refs. [52, 53]. The asymptotic expressions for  $\Phi_V$  and  $\Phi_S$  obtained below (see eqs. (4.6), (4.10), and (4.11)) are also in accordance with those found in refs. [52, 53]. As mentioned above, the Dirichlet boundary condition at  $u = 0$  imposed on  $Z_T^{in}(u)$  requires that  $\mathcal{A}_T(\mathfrak{w}, \mathfrak{q}) = 0$ , which is the same as the condition one obtains by imposing Dirichlet boundary condition on the RWZ variable  $\Phi_T^{in}(u)$ . Namely, the relations (4.1), (4.3) and (4.4) imply in  $\mathcal{A}_T(\mathfrak{w}, \mathfrak{q}) = \mathcal{C}_T(\mathfrak{w}, \mathfrak{q}) = 0$ . Now since the equation  $\mathcal{A}_T(\mathfrak{w}, \mathfrak{q}) = 0$  furnishes the spectrum of the QNM one concludes that the spectra of the tensorial gravitational QNM obtained by using KS or RWZ variables are identical for all dimensions  $d > 4$ .

#### 4.2 Vectorial sector

In the case of gravitational vectorial perturbations, we can show that the KS and RWZ variables are related by

$$Z_V(u) = f u^{d-2} \frac{\partial}{\partial u} \left( u^{-\frac{d-2}{2}} \Phi_V(u) \right). \quad (4.5)$$

Now eqs. (3.7) and (3.13) yield the following asymptotic form for the solution  $\Phi_V^{in}(u)$  which satisfy an incoming-wave condition at the horizon:

$$\Phi_V^{in}(u) = \mathcal{C}_V(\mathfrak{w}, \mathfrak{q}) u^{-\frac{d-4}{2}} + \dots + \mathcal{D}_V(\mathfrak{w}, \mathfrak{q}) u^{\frac{d-2}{2}} + \dots. \quad (4.6)$$

Therefore, eqs. (4.1), (4.5) and (4.6), and the Dirichlet boundary condition at  $u = 0$  imposed on  $Z_V^{in}(u)$  imply in  $\mathcal{A}_V(\mathfrak{w}, \mathfrak{q}) = -(d-3)\mathcal{C}_V(\mathfrak{w}, \mathfrak{q}) = 0$ , from what one concludes that the KS and the RWZ quasinormal spectra are identical to each other for all dimensions  $d \geq 4$ .

### 4.3 Scalar sector

For the scalar gravitational sector we were not able to find a simple relation between  $Z_S(u)$  and  $\Phi_S(u)$ , and then the analysis becomes more evolved than for the other sectors. After some algebra we find

$$\begin{aligned}
 2(d-2) [2\mathfrak{q}^2 u - (d-2) f']^2 Z_S(u) = & \left\{ -2(d-2)^3 f'^2 \mathfrak{w}^2 \right. \\
 & + (d-2) \left[ (d-2) g(u) u^{d-3} + 4 h(u) \mathfrak{q}^2 \right] \mathfrak{q}^2 \\
 & \left. + 4 \left[ 2(d-2) - (d-3) u^{d-1} \right] u^2 \mathfrak{q}^6 \right\} u^{\frac{d-2}{2}} \Phi_S(u) \\
 & + (2\mathfrak{q}^2 u - (d-2) f') [(d-1)(1-f) - 4(d-2)(d-3)f^2] \mathfrak{q}^2 u^{\frac{d-2}{2}} \Phi'_S(u),
 \end{aligned} \tag{4.7}$$

where the coefficients  $h(u)$  and  $g(u)$  are defined by

$$\begin{aligned}
 h(u) = & 2 + (d-2)(d-5) - [3 + (d-10)(d-2)] u^{d-1} \\
 & + (d-4)(d-3) u^{2(d-1)} - 2u^2 \mathfrak{w}^2,
 \end{aligned} \tag{4.8}$$

$$\begin{aligned}
 g(u) = & 2 [1 + (d-2)^2 + 2(d-2)^3] u^{d-1} - (d-3)^2 (d-1) u^{2(d-1)} \\
 & - 2(d-1) [(d-2)(d-3) + 4u^2 \mathfrak{w}^2].
 \end{aligned} \tag{4.9}$$

Using eqs. (3.7) and (3.15) for the RWZ scalar variable  $\Phi_S(u)$  we find the following asymptotic form for the incoming-wave solution at the horizon  $\Phi_S^{in}$ :

$$\Phi_S^{in}(u) = \mathcal{C}_S(\mathfrak{w}, \mathfrak{q}) u^{-\frac{d-6}{2}} + \dots + \mathcal{D}_S(\mathfrak{w}, \mathfrak{q}) u^{\frac{d-4}{2}} + \dots, \quad d \neq 5, \tag{4.10}$$

$$\Phi_S^{in}(u) = [\mathcal{C}_S(\mathfrak{w}, \mathfrak{q}) + \dots + \mathcal{D}_S(\mathfrak{w}, \mathfrak{q}) \ln u + \dots] \sqrt{u}, \quad d = 5, \tag{4.11}$$

where  $\mathcal{C}_S$  and  $\mathcal{D}_S$  are the connection coefficients associated to eq. (3.7).

Since the asymptotic forms of  $Z_S^{in}$  and  $\Phi_S^{in}$  critically depend on the number of dimensions we analyze the cases  $d = 4$ ,  $d = 5$  and  $d > 5$  separately.

#### 4.3.1 Four dimensions

In the four-dimensional case ( $d = 4$ ), eqs. (4.7) and (4.10), and the Dirichlet boundary condition on  $Z_S^{in}(u)$  lead to the condition

$$\mathcal{C}_S(\mathfrak{w}, \mathfrak{q}) + \frac{3\mathcal{D}_S(\mathfrak{w}, \mathfrak{q})}{\mathfrak{q}^2} = 0. \tag{4.12}$$

In terms of the RWZ variable  $\Phi_S^{in}(u)$ , this is a boundary condition of Robin type, i.e., a mixing between Dirichlet and Neumann boundary conditions. Therefore, in order for both of the spectra being the same, and in order for the QN frequencies being given by the poles of the dual stress-energy tensor correlator, one must impose Dirichlet boundary condition on  $Z_S^{in}(u)$  at  $u = 0$  and Robin boundary condition on  $\Phi_S^{in}(u)$  at  $u = 0$ . This result explains why Dirichlet boundary conditions imposed on  $Z_S^{in}(u)$  and  $\Phi_S^{in}(u)$  lead to different quasinormal spectra. In ref. [53] it was argued that the non-deformation of the boundary metric favors a Robin condition on the master field  $\Phi_S(u)$ , and using such a boundary

condition they have found the hydrodynamic QNM of the scalar gravitational sector in the  $d = 4$  Schwarzschild-AdS spacetime. Our result is consistent with that analysis, since both of the results are identical for large  $r_h/R$ , a regime where the Schwarzschild-AdS black hole reduces to the AdS black brane. As a matter of fact, it can be shown that for a spacetime in which the subspace  $\mathcal{K}^2$  has constant curvature  $K$  and the event horizon is such that  $r_h \gg R$ , the relation (4.12) is replaced by  $\mathcal{C}_S + 3\mathcal{D}_S/(\mathfrak{q}^2 - 2KR^2/r_h^2) = 0$ , which reproduces our result for  $K = 0$ , and the result of ref. [53] for  $K = 1$  and  $\mathfrak{q}^2 = l(l+1)R^2/r_h^2$ , where  $l$  is the angular momentum of the perturbation.

### 4.3.2 Five dimensions

The asymptotic form for  $\Phi_S^{in}(u)$  for  $d = 5$  is given by eq. (4.11). The Dirichlet boundary condition on  $Z_S^{in}(u)$  at  $u = 0$ , together with eqs. (4.7) and (4.11), furnishes the following condition

$$\mathcal{D}_S(\mathfrak{w}, \mathfrak{q}) = 0. \tag{4.13}$$

This is equivalent to the condition of not changing the metric on the AdS boundary, as shown in ref. [52] in the case of a spherically symmetric AdS<sub>5</sub> black hole. Hence, using eqs. (4.13) and (4.11) it is found that the RWZ variable  $\Phi_S(u)$  and the KS variable  $Z_S(u)$  yield the same quasinormal spectrum in five-dimensional spacetimes as soon as one imposes the condition  $\Phi_S^{in}/\sqrt{u} - \mathcal{C}_S = 0$  at infinity ( $u \rightarrow 0$ ).

### 4.3.3 Six and higher dimensions

Following the same procedure as for  $d = 4$  and  $d = 5$  above, Dirichlet boundary condition  $Z_S^{in}(u = 0) = 0$  and eqs. (4.7) and (4.10) yield

$$(d-3)(d-5)\mathcal{C}_S(\mathfrak{w}, \mathfrak{q}) = 0, \tag{4.14}$$

from what we conclude that for  $(d-3)(d-5) \neq 0$  the quasinormal spectra furnished by the master variables  $Z_S(u)$  and  $\Phi_S(u)$  are identical.

It is worth stressing here the relevance of the above results. They allow us to use the most convenient gauge-invariant equations for each specific case. For instance, whenever one has any kind of difficulty in finding QN frequencies with a certain set of equations based, say, on the KS gauge-invariant variables, one can try the other set of equations, based on the RWZ variables. Moreover, some numerical methods require Schrödinger-like equations such as in the case of the time-evolution method used in the present work, as we will see in section 6.

## 5 The quasinormal spectra: analytical results

In this section we report on the procedure for calculating the QNM in some asymptotic limits where results can be expressed in closed form. In particular, the hydrodynamic limit of the QNM dispersion relations are obtained analytically. A brief analysis of the results is given for each sector with calculations done considering an arbitrary number of spacetime dimensions  $d$ . Other asymptotic regions of the QNM spectra such as large frequencies and large spacetime dimensions are also analyzed.

## 5.1 The hydrodynamic limit

The hydrodynamic limit is the regime in which  $\omega$  and  $k$  are sufficiently smaller than the Hawking temperature  $T$ , i.e.,  $\mathfrak{w}, \mathfrak{q} \ll 1$ . In such a regime it is possible to express the solutions of the perturbation equations in the form of power series in  $\mathfrak{w}$  and  $\mathfrak{q}$ . By keeping just the lowest order terms one finds the so-called hydrodynamic limit of the dispersion relations ( $\mathfrak{w} \rightarrow 0, \mathfrak{q} \rightarrow 0$ ). Such a procedure is well known in the literature, and we do not reproduce it here. The hydrodynamic limit of the dispersion relations to first order approximation are known for some particular number of dimensions. For instance, the vectorial and scalar sectors with  $d = 4, 7$ , were treated in [15, 16], and in five dimensions the topic was explored in refs. [13, 14, 51, 56]. This limit of the QNM spectra to second order approximation for  $d = 4, 5, 7$  has been studied in refs. [34–36]. Here we show the results to first order approximation for all spacetime dimensions and for all sectors of metric perturbations. We work with the KS gauge-invariant variables and Dirichlet boundary condition at  $u = 0$ .

### 5.1.1 Tensorial perturbations

In the limit of small frequencies and small wavenumbers we find the solution to eq. (3.18), satisfying the condition of representing ingoing waves at the horizon, as

$$Z_T^{in}(u) = \mathcal{C}_T f^{i\mathfrak{w}/(d-1)} [1 + \mathcal{O}(\mathfrak{w}^2)], \quad (5.1)$$

where  $\mathcal{C}_T$  is an arbitrary normalization constant. Imposing the Dirichlet condition at anti-de Sitter boundary  $u = 0$ , namely  $Z_T^{in}(0) = 0$ , and noting that  $f(0) = 1$ , it follows that there is no solution to eq. (3.18) satisfying the QNM boundary conditions and being also a hydrodynamic QNM (i.e., satisfying  $|\mathfrak{w}| \ll 1$  and  $|\mathfrak{q}| \ll 1$ ). The non-existence of tensorial hydrodynamic QNM is compatible with the expectations from hydrodynamics [56].

### 5.1.2 Vectorial perturbations

The first order perturbative solution to eq. (3.20) satisfying the condition of representing an ingoing wave at the horizon is given by

$$Z_V^{in} = \mathcal{C}_V f^{i\mathfrak{w}/(d-1)} \left[ 1 - \frac{i \mathfrak{q}^2 f}{(d-1) \mathfrak{w}} + \mathcal{O}(\mathfrak{w}^2) \right], \quad (5.2)$$

with  $\mathcal{C}_V$  being a normalization constant. The Dirichlet boundary condition at infinity,  $Z_V^{in}(0) = 0$ , implies the following dispersion relation:

$$\mathfrak{w} = \frac{i}{d-1} \mathfrak{q}^2 + \mathcal{O}(\mathfrak{q}^3). \quad (5.3)$$

The dispersion relation (5.3) can be interpreted in terms of traveling waves in non-ideal fluids. In fact, it is expected from hydrodynamics that a transversal momentum fluctuation presents a shear mode, corresponding to a purely damped mode with dispersion relation [76]

$$\mathfrak{w} = i \frac{4\pi T}{d-1} D \mathfrak{q}^2, \quad (5.4)$$



with  $D$  being a diffusion constant carrying dimensions of length. Therefore, the result in eq. (5.3) agrees with hydrodynamics and the quasinormal frequency can be interpreted as the dispersion relation for the shear mode, with diffusion constant  $D = 1/4\pi T$ .

Finally, it is worth noticing that relation (5.3) holds for gravitational perturbations of plane-symmetric black holes in asymptotically AdS spacetimes of any dimension  $d \geq 4$ , and it is in agreement with previous results for  $d = 4, 5, 7$  (see [13, 15, 37, 38, 51, 56]).

### 5.1.3 Scalar perturbations

Solving eq. (3.22) perturbatively in a power series in  $\mathfrak{w}$  and  $\mathfrak{q}$  yields

$$Z_S^{in} = \mathcal{C}_S f^{i\mathfrak{w}/(d-1)} \left\{ \left[ 2 - 2(d-2) \frac{\mathfrak{w}^2}{\mathfrak{q}^2} - (d-3)(f-1)^2 \right] + \frac{4i\mathfrak{w}(d-3)f}{(d-1)} + \mathcal{O}(\mathfrak{w}^2) \right\}, \tag{5.5}$$

with  $\mathcal{C}_S$  being an integration (normalization) constant. Imposing the Dirichlet boundary condition at  $u = 0$  on  $Z_S^{in}(u)$ , and taking into account we are working in the hydrodynamic limit, we obtain

$$\mathfrak{w} = \pm \frac{\mathfrak{q}}{\sqrt{d-2}} + \frac{(d-3)i}{(d-2)(d-1)} \mathfrak{q}^2 + \mathcal{O}(\mathfrak{q}^3). \tag{5.6}$$

In order to compare the above result (5.6) to hydrodynamics we first observe that for a conformal field theory the energy-momentum tensor is traceless, so that the energy density  $\varepsilon$  and the pressure  $P$  of the dual plasma are related by  $\varepsilon = (d-2)P$  and, consequently, the speed of sound in the medium is  $v_s = |\partial P / \partial \varepsilon|^{1/2} = 1/\sqrt{d-2}$ . Thus, the expected dispersion relation for the longitudinal momentum fluctuations, in the hydrodynamic limit, must correspond to the sound wave mode [76]

$$\mathfrak{w} = \pm v_s \mathfrak{q} + \frac{4\pi i (d-3)T}{(d-2)(d-1)} D \mathfrak{q}^2. \tag{5.7}$$

The constant  $D$  in eq. (5.7) is the same diffusion constant appearing in eq. (5.4). In fact, comparing eqs. (5.6) and (5.7) we find  $D = 1/4\pi T$ , agreeing with the value found from the analysis of the hydrodynamic limit of vectorial perturbations. This shows that the result given in eq. (5.6) is consistent with the expected result from hydrodynamics. Furthermore, this result is also in agreement with the previous results in the literature for  $d = 4, 5, 7$  (see [14, 16, 35, 37, 39, 51, 56]).

## 5.2 Asymptotic analysis of the QNM

### 5.2.1 Small wavenumbers, large frequencies

There is an alternative analysis for large frequencies with finite wavenumbers, namely  $\mathfrak{w} \gg \mathfrak{q}$ . To first order approximation such a condition is equivalent to the asymptotic limit  $\mathfrak{q} \rightarrow 0$ , as far as all the other parameters of the model are kept fixed. That is to say, taking the limit  $\mathfrak{w} \rightarrow \infty$  with fixed  $\mathfrak{q}$  yields the same approximate equation as taking the limit  $\mathfrak{q} \rightarrow 0$  with finite  $\mathfrak{w}$ . For all of the perturbation equations (3.18), (3.20) and (3.22) with a little algebra we find

$$Z_p'' - \left[ \frac{d-1-f}{uf} \right] Z_p' + \frac{\mathfrak{w}^2}{f^2} Z_p = 0, \tag{5.8}$$

where  $p$  denotes the perturbative sector, as already indicated. It is obvious that eq. (5.8) necessarily imply in identical non-hydrodynamic quasinormal frequencies at  $\mathfrak{q} = 0$  for all of the perturbation types. The same result was also found in our numerical calculations, as it will be seen in the next section (see table 4). With this result we conclude that the dispersion relations for large frequencies are the same for all the three perturbation sectors of a black brane, a result which was already obtained by Natário and Schiappa [77] for the Schwarzschild-AdS (Kottler) solution.

### 5.2.2 Large number of spacetime dimensions

In this section we analyze the perturbation equations when the number of spacetime dimensions is large, namely  $d \rightarrow \infty$  with finite  $\mathfrak{w}$  and  $\mathfrak{q}$ . For simplicity, in this analysis we consider the master equations for the RWZ gauge-invariant variables (eq. (3.7)), in which case the analysis reduces to investigate the asymptotic form of the potentials (3.10), (3.13) and (3.15) in the limit  $d \gg 4$ . We thus find

$$V_T \rightarrow \frac{d^2}{4u^2} f \left( 1 + u^{d-1} \right), \tag{5.9}$$

$$V_V \rightarrow \frac{d^2}{4u^2} f \left( 1 - 3u^{d-1} \right), \tag{5.10}$$

$$V_S \rightarrow \frac{d^2}{4u^2} f \left( 1 + u^{d-1} \right). \tag{5.11}$$

It is seen that in such a limit the tensorial and scalar potentials are the same. Moreover, in the intervening region between the AdS boundary and the horizon ( $0 < u < 1$ ), the second term of the above expressions within the parentheses tend to zero in the limit  $d \rightarrow \infty$ , so that the potentials are identical in this region. Moreover, the tensorial, scalar and vectorial potentials approach the same values at the boundaries, namely  $\lim_{u \rightarrow 0} V_p = d^2/4u^2$  and  $\lim_{u \rightarrow 1} V_p = 0$ . These results suggest that the QNM spectra of the three perturbation sectors for large  $d$  are identical. This is an important result because it shows the isospectrality of the gravitational QNM of higher-dimensional AdS black holes. Let us observe that this cannot be seen in our graphs because our values of  $d$  are not large enough when compared to the other parameters, in particular  $d \sim 4$  in our numerical results.

## 6 Numerical results

### 6.1 Methods

We use two different methods to determine the gravitational QNM frequencies of the black branes in the spacetime (2.1). The first one is a series expansion method [74, 78], which reduces the problem to finding roots of a polynomial. The second method employed in this work consists on a direct time-evolution of the gravitational perturbations in these backgrounds [79, 80].

#### 6.1.1 Power series method

The method developed by Horowitz and Hubeny [78] consists in expanding the Fourier transformed perturbation variables in power series of  $u$  around the event horizon,  $u = 1$ .

The condition of ingoing wave at the horizon is imposed on each perturbation function. More specifically, the first step is to expand each of the functions  $Z_p(u)$ ,  $p = T, V, S$  in a Frobenius series of the form  $Z_p(u) = (1 - u)^{i\mathfrak{w}/(d-1)} \sum_j a_j(\mathfrak{w}, \mathfrak{q})(1 - u)^j$ . The Dirichlet boundary condition at infinity is then imposed, and we obtain an equation in the form of an infinite sum for the coefficients,

$$\sum_{j=0}^{\infty} a_j(\mathfrak{w}, \mathfrak{q}) = 0, \tag{6.1}$$

the roots of which yield the dispersion relation  $\mathfrak{w} = \mathfrak{w}(\mathfrak{q})$ . During the calculation process, the infinite sum (6.1) is truncated at a sufficiently large number of terms and then one finds the roots of a polynomial in  $\mathfrak{w}$ . The accuracy of the results is then verified through the relative variation between the roots of two successive partial sums. The roots so obtained are the quasinormal frequencies, which we write as

$$\mathfrak{w} = \mathfrak{w}_R + i \mathfrak{w}_I. \tag{6.2}$$

Even though the method developed by Horowitz and Hubeny [78] is well suited to large AdS black holes and black branes, it is found that the convergence properties worsen for large wavenumbers. Moreover, the capability of the Horowitz-Hubeny method in finding the QN frequencies depends in an unclear way on the variables chosen, on the considered region of the spectrum one seeks for solutions and on the spacetime dimension. For instance, by using the master equation for the KS variables, this method produced dispersion relations of vectorial modes only for  $\mathfrak{q} < 4$ . Then, by shifting to the master equation for the RWZ variables we were able to find satisfactory results for larger wavenumbers, at least for  $d = 4, 5$  and 6. For higher spacetime dimensions convergence problems occur for all perturbation sectors. In particular, for the tensorial and scalar sectors via the master equations with KS variables the numerical convergence problems of the series solutions arise for dimensions larger than six ( $d > 6$ ), and higher overtones ( $n > 1$  or 2), even for intermediate wavenumber values ( $\mathfrak{q} \sim 1$ ). Because of these convergence problems we used this method to compute the dispersion relations for the first five quasinormal modes for each perturbation sector, only for  $d = 4, 5$  and 6. In higher dimensions we used a different method, a time-domain evolution method, which allows one to read off the fundamental QNM for each sector, as seen in the following, directly from the decay timescale and ringing frequency of the signal.

### 6.1.2 Time evolution method

The time evolution approach employed in the present work is based on a characteristic initial value formulation of the perturbation wave equations [79, 81, 82]. The time-domain versions of the equations (3.7) are rewritten in terms of the normalized light-cone variables  $w = r_h t/R^2 - \mathfrak{r}_*$  and  $v = r_h t/R^2 + \mathfrak{r}_*$ . The wave equations are integrated numerically

using the finite difference scheme introduced in [80],

$$\left[1 - \frac{\Delta^2}{16} V_p(S)\right] \Phi_p(N) = \Phi_p(E) + \Phi_p(W) - \Phi_p(S) - \frac{\Delta^2}{16} [V_p(S)\Phi_p(S) + V_p(E)\Phi_p(E) + V_p(W)\Phi_p(W)] . \tag{6.3}$$

The scalar, vectorial and tensorial sectors are indexed by  $p$ . The points  $N$ ,  $S$ ,  $W$  and  $E$  are defined as:  $N = (w + \Delta, v + \Delta)$ ,  $W = (w + \Delta, v)$ ,  $E = (w, v + \Delta)$  and  $S = (w, v)$ . The discretization step  $\Delta$  is a function of the grid size and number of point in the grid [79].

Initial data are specified on the null surfaces  $w = w_0$  and  $v = v_0$ . Since the behavior of the integrated wave functions is largely insensitive to the choice of initial data (which was empirically verified in the present scenario), we set  $\Phi_p(w, v = v_0) = 0$  and use a Gaussian pulse as initial perturbation  $\Phi_p(w = w_0, v) = \exp[-(v - v_c)^2/2\sigma^2]$ . After the integration is completed, the values of  $\Phi_p$  on selected curves are extracted. The quasinormal fundamental frequency can (usually) be accurately estimated from the data.

The algorithm precision depends on the number of points and size of the discretized grid. One basic requirement for the method is the convergence of the code with respect to the variation of the number of grid points. It was observed in our numerical experiments that the convergence rate varied with the parameters of the system, with the worse performance in the small  $q$  limit for the tensorial and scalar sectors. Since in this limit the power series method is usually reliable, and the concordance of both methods is very good in a wide range of parameter space, it is accurate to say that the methods employed in this work are complementary.

In the following we show some numerical results and analyze in some detail the dispersion relations of the fundamental mode, i.e., the QNM with the smallest imaginary part of the frequency, for each perturbation sector. As we see below, for the vectorial and scalar sectors, the fundamental modes in the low wavenumber regime are in fact hydrodynamic QNM, so denominated because they present a characteristic behavior in the hydrodynamic limit,  $\omega \rightarrow 0$  when  $q \rightarrow 0$ . The basic motivation for studying the hydrodynamic modes is because they are important modes in connection to the AdS/CFT correspondence, since they furnish (for low  $q$  values) the thermalization time in the conformal field theory at AdS spatial infinity.

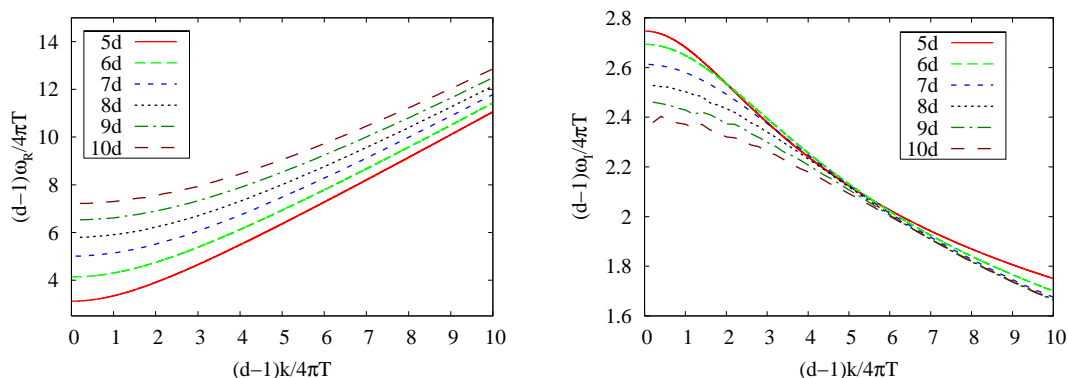
## 6.2 Numerical results for the tensorial QNM

In table 1 we list the values obtained for the QN frequencies of the first five modes (overtone numbers  $n = 0, 1, \dots, 4$ ), with  $q = 0$  in five and six spacetime dimensions. From the results shown in this table, it is seen that the fundamental equation for the tensorial gravitational perturbations (3.18) for  $d = 5$  and the use of the Horowitz-Hubeny method reproduce the results in the literature [83] with very good accuracy. For six dimensions, we have not found any previous result in the literature for comparison.

The dispersion relations for the fundamental tensorial QNM for  $d = 5, 6, \dots, 10$  are shown in figure 1. The Horowitz-Hubeny method yielded the results for  $d = 5, 6$  and for small wavenumbers in  $d = 7$ . The time-evolution method was used to compute the

$n$	$d = 5$		$d = 6$	
	$\omega_R$	$\omega_I$	$\omega_R$	$\omega_I$
0	3.11945	2.74668	4.13591	2.69339
1	5.16952	4.76357	6.60919	4.45349
2	7.18793	6.76957	9.02574	6.19321
3	9.19720	8.77248	11.4241	7.92685
4	11.2027	10.7742	13.8136	9.65854

**Table 1.** The first five tensorial QNM with zero wavenumber for a plane-symmetric AdS black hole (black brane) in five and six spacetime dimensions.

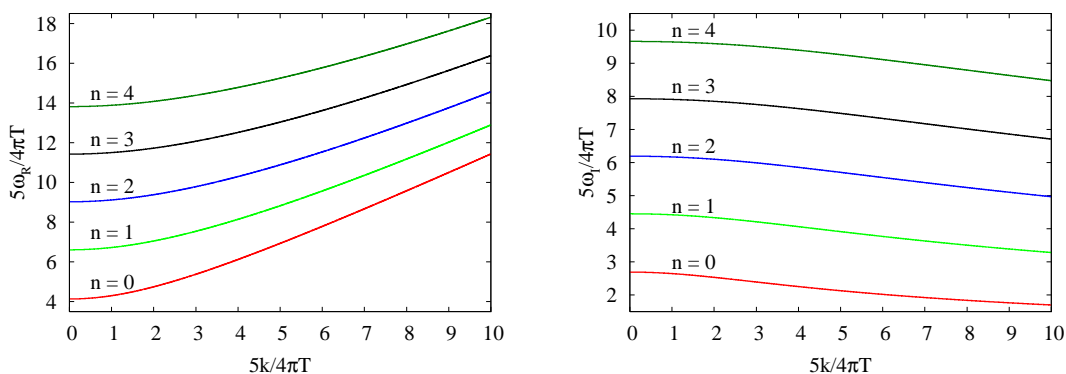


**Figure 1.** Dispersion relations for the first tensorial QNM of AdS black branes in several dimensions,  $d = 5, 6, \dots, 10$ , as indicated. The small wiggles in the curves of the imaginary parts of the frequencies for larger values of  $d$  indicate numerical convergence problems (see the text).

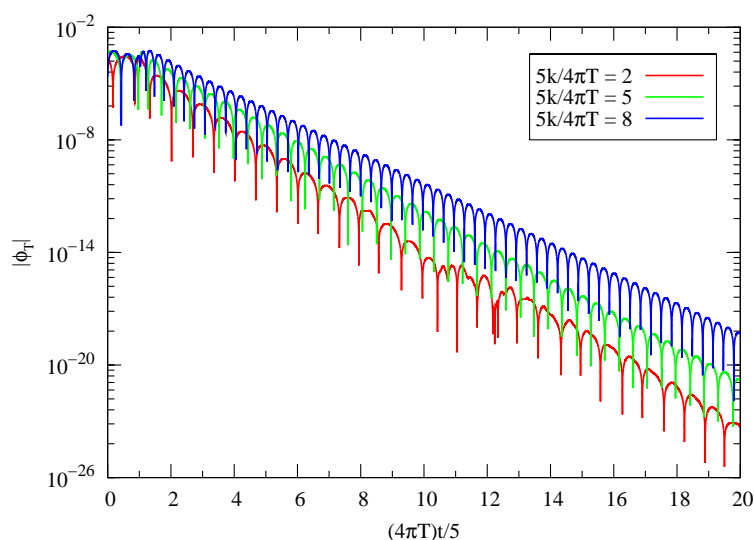
dispersion relations for the other dimensions and for large wavenumbers in  $d = 7$ . We observe some convergence problems for small values of  $q$  and  $d > 7$ , as it is apparent from the dispersion relation curves for the imaginary part of the frequency (see the right panel of figure 1).

It is also apparent in figure 1 that the real and imaginary parts of the QNM frequencies present an overall behavior that appears to be independent of the number of spacetime dimensions. Dispersion relations  $\omega_R(q)$  approaching straight lines and dispersion relations  $\omega_I(q)$  approaching zero as the wavenumber  $q$  increases. The same feature is also shown for higher overtones in figure 2. However, the larger the overtone index  $n$  and/or the larger the number of dimensions, the faster the frequency  $\omega_I$  approaches zero. This is in agreement with the results by Festuccia and Liu [61] (see also ref. [62]). In fact, it was shown in [61] by means of analytical methods, and confirmed in [62] through numerical methods, that the dispersion relations for large wavenumbers are given approximately by  $\omega_R = q + \alpha_R q^{-\beta}$  and  $\omega_I = \alpha_I q^{-\beta}$ , where  $\alpha_{R,I}$  are parameters depending on the number of dimensions and overtone index, and  $\beta = (d - 3)/(d + 1)$ .

In figure 2 we have the dispersion relations for the first five tensorial modes in six dimensions where we can see that for a finite fixed temperature, the imaginary parts of the frequencies decrease with the wavenumber value, while the real parts increase with the



**Figure 2.** Dispersion relations for the first five tensorial QNM of AdS black branes in  $d = 6$ .



**Figure 3.** Time-evolution profiles of the tensorial mode perturbations for selected values of  $q$  in  $d = 6$  (from top to bottom, for  $q = 8, 5, 2$ ).

wavenumber. The behavior is qualitatively similar for  $d = 5$ .

Time-evolution profiles of the tensorial perturbations can be seen in figure 3. The oscillatory decay, which is characteristic of a QNM with  $\mathfrak{w}_R \neq 0$ , dominates the intermediate- and late-time behavior of the wave function  $\Phi_T$ . In contrast with what happens in asymptotically flat spacetimes, our numerical results for all perturbation sectors show no sign of a power-law tail at late stages, in agreement with earlier predictions [78, 84].

As expected from the study of the hydrodynamic limit of tensorial quasinormal modes given in section 5.1.1, the numerical analysis of eq. (3.18) based on the Horowitz-Hubeny approach found no QNM satisfying simultaneously the conditions  $\mathfrak{w} \ll 1$  and  $q \ll 1$ .

### 6.3 Numerical results for the vectorial QNM

The frequencies for the hydrodynamic and for the first five non-hydrodynamic vectorial QNM are shown in table 2, where we have set  $d = 5, 6$  and  $q = 2$ . The results for five

$n$	$d = 5$		$d = 6$	
	$\mathfrak{w}_R$	$\mathfrak{w}_I$	$\mathfrak{w}_R$	$\mathfrak{w}_I$
0	0	1.19612	0	0.87233
1	3.51823	2.58319	4.51460	2.57492
2	5.46616	4.66081	6.88034	4.37633
3	7.43187	6.69069	9.24255	6.13420
4	9.40729	8.70698	11.6070	7.87837
5	11.3889	10.7175	13.9739	9.61647

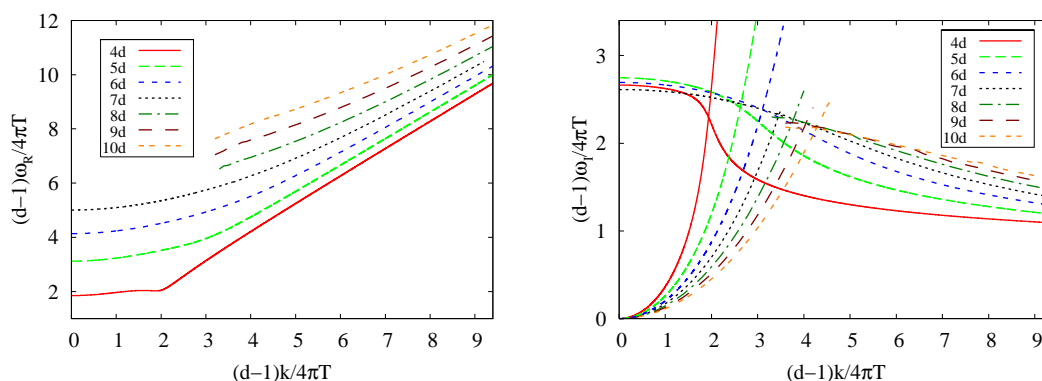
**Table 2.** Some data for the frequencies of the vectorial gravitational QNM of five and six dimensions for  $q = 2$ .

dimensions coincide with those of ref. [56]. As far as we are aware of, there are no data in the literature for comparison to the results shown in table 2 for six dimensions.

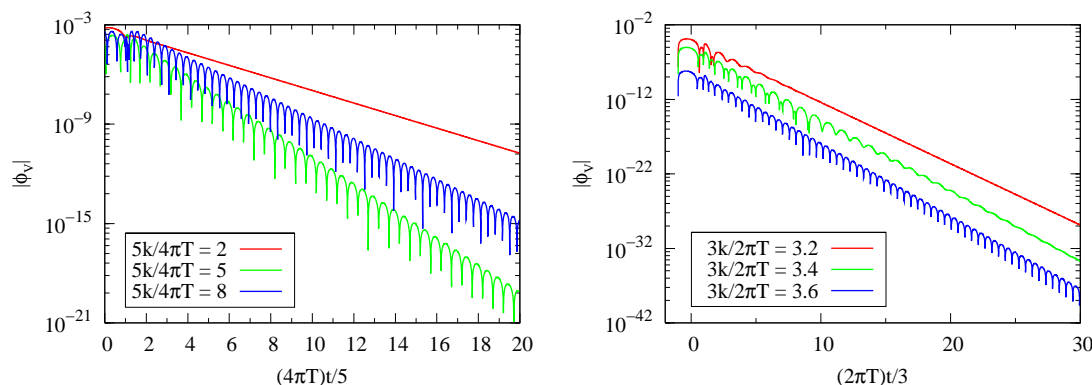
In figure 4 we plot the hydrodynamic and the first non-hydrodynamic vectorial QNM for several dimensions,  $d = 4, 5, \dots, 10$ , as indicated (recall that the hydrodynamic modes are those for which the frequencies  $\mathfrak{w}(q)$  vanish when  $q \rightarrow 0$ ). The numerical results confirm the fact that the hydrodynamic vectorial QNM are purely damped modes, and so the dispersion relation curves have just the imaginary parts, as shown in the right panel of figure 4. Since we could not find the QNM using the Horowitz-Hubeny method for  $d > 7$ , the dispersion relations for  $d = 8, 9, 10$  were obtained through a time-domain evolution method, as described in section 6.1.2, so the curves for the non-hydrodynamic QNM at low values of  $q$  could not be found, because the hydrodynamic mode dominates in that region. This is clearly seen in the left panel of figure 4, where the dispersion relation curves for the first non-hydrodynamic mode at small values of  $q$  and  $d = 8, 9, 10$  are missing. As a matter of fact, the curves for  $d = 7$  were obtained by joining the results from both of the numerical methods used here. We see that the dispersion relations for  $d = 5, 6, 7$  have all the same behavior. A slight difference is observed in the curve  $\mathfrak{w}_R \times q$  for  $d = 4$  (see the lowest curve of the left panel in figure 4) in which it is seen a “knee” around  $q \simeq 2$ . This local minimum in the real part of the frequency is present in all of the gravitational vectorial modes of the four-dimensional black brane [51]. As seen from figure 4, such a local minimum disappears in higher dimensions.

Typical time-domain evolution profiles of the vectorial QNM are presented in figure 5. The transition from the hydrodynamic shear-mode regime of perturbations to the ordinary-QNM regime appears in the time evolution of  $\Phi_V$  as a transition from non-oscillatory (for small  $q$ ) to oscillatory (large  $q$ ) late-time decay. The left panel of figure 5 exploits exactly this feature of the vectorial gravitational QNM in  $d = 6$ : non-oscillatory time-evolution for  $q = 2$  and oscillatory time-evolution for  $q = 5, 8$ . This transition is important to the CFT side of the AdS/CFT correspondence, since it is interpreted as the hydrodynamic-to-collisionless crossover which is expected to arise in generic systems [49, 85]. The right panel of figure 5, now for seven-dimensional black branes, shows the mode mixing as seen in the two upper curves of such a figure. The choice of values for  $d$  and  $q$  was made with the aim of showing not only the transition from a non-oscillatory to an oscillatory regime,





**Figure 4.** Dispersion relations for the first non-hydrodynamic vectorial QNM of the AdS black brane in several dimensions,  $d = 4, 5, \dots, 10$ . The curves for the hydrodynamic modes are also shown in the right panel for  $0 \leq q \lesssim 4$ .

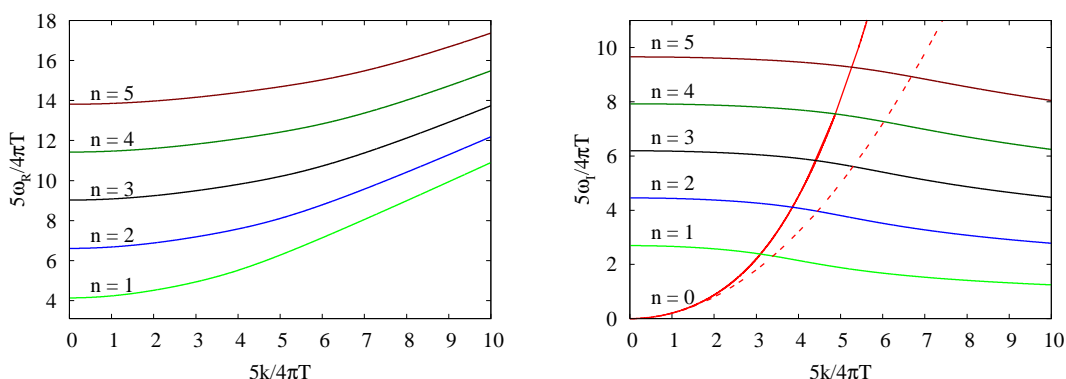


**Figure 5.** Time-evolution profiles of the vectorial mode perturbations for  $d = 6$  (left panel) and  $d = 7$  (right panel). In the left panel we plot (from top to bottom) the curves for  $q = 2, 8, 5$  wavenumbers. The transition from non-oscillatory ( $q = 2$ ) to oscillatory late-time decay ( $q = 5, 8$ ) is clearly seen. In the right panel, from top to bottom, we see the curves for  $q = 3.2, 3.4, 3.6$ . The mode mixing is apparent.

but also to exploit the mode mixing feature of the QNM. This feature can be understood by assuming that for intermediate times an oscillatory mode dominates, while for later times a non-oscillatory mode dominates. In such a situation, the identification of a specific QNM frequency from the numerical data is very difficult. This explains the missing parts of the dispersion relation curves for  $d = 7, 8, 9, 10$  in figure 4.

The graphs in figure 6 show the first five vectorial non-hydrodynamic quasinormal modes for the six-dimensional black branes. The hydrodynamic QNM and the shear mode of equation (5.3) are also shown in the right panel of this figure. We can see that the behavior of the non-hydrodynamic modes is very similar to the tensorial QNM (cf. figure 2). Similarly to the tensorial sector, the higher overtone modes ( $n > 1$ ) follow the overall behavior of the first non-hydrodynamic QNM. Among the most prominent differences between vectorial and tensorial QNM, we see that the imaginary parts of the frequencies approach zero with growing  $q$  faster in the vectorial case than in the tensorial case, which means





**Figure 6.** Dispersion relations for the hydrodynamic mode and for the first five vectorial non-hydrodynamic QNM of the AdS black brane in six dimensions. The dashed line in the right panel corresponds to the shear mode of equation (5.3).

$n$	$d = 5$		$d = 6$	
	$\omega_R$	$\omega_I$	$\omega_R$	$\omega_I$
0	1.48286	0.57256	1.19804	0.58389
1	3.46702	2.68602	4.40613	2.62487
2	5.41108	4.71412	6.79949	4.40716
3	7.37878	6.72773	9.17633	6.15701
4	9.35747	8.73596	11.5503	7.89671
5	11.3422	10.7416	13.9214	9.63703

**Table 3.** The frequencies of the hydrodynamic and the first five non-hydrodynamic scalar QNM in five- and six-dimensional spacetimes, with  $\eta = 2$ .

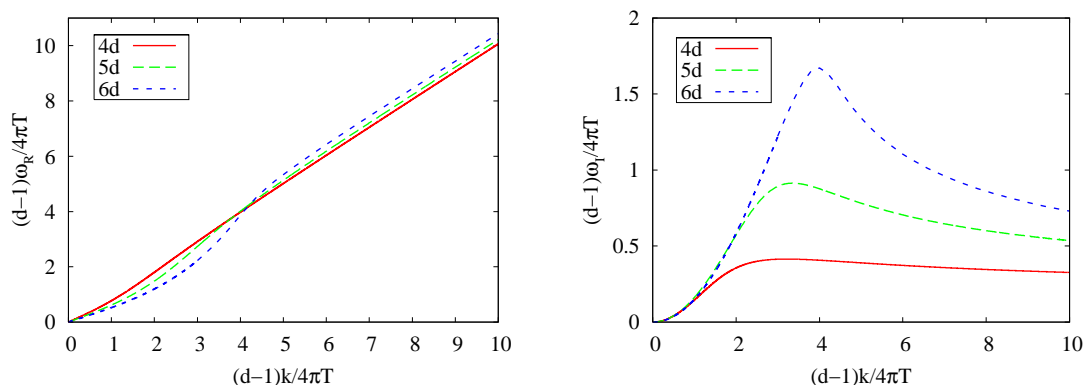
that the thermalization time in the dual CFT is dominated by the vectorial perturbations in comparison to the corresponding tensorial modes.

The numerical results shown in figures 4 and 6 agree with the analytical relations for small frequencies and wavenumbers (cf. right panel of figure 6), what corresponds to the hydrodynamic limit of the QNM spectrum. The behavior of the hydrodynamic mode is very important for the interpretation of quasinormal modes in terms of the AdS/CFT correspondence since it furnishes the value of the diffusion constant  $D$  and dominates the thermalization timescale of the perturbations for small wavenumbers. The study of the thermalization time is one of the subjects of section 7.

### 6.4 Numerical results for the scalar QNM

In table 3 we list the hydrodynamic and the first five non-hydrodynamic scalar QNM of the plane-symmetric AdS black holes in five and six spacetime dimensions, fixing  $\eta = 2$ . Again, our results agree with those of ref. [56] for the five-dimensional case, while in the six-dimensional case we have not found similar data in the literature for comparison.

We have used the Horowitz-Hubeny method to obtain the complete dispersion relations of the dominant scalar quasinormal modes for  $d = 4, 5$ , and 6. These results are shown



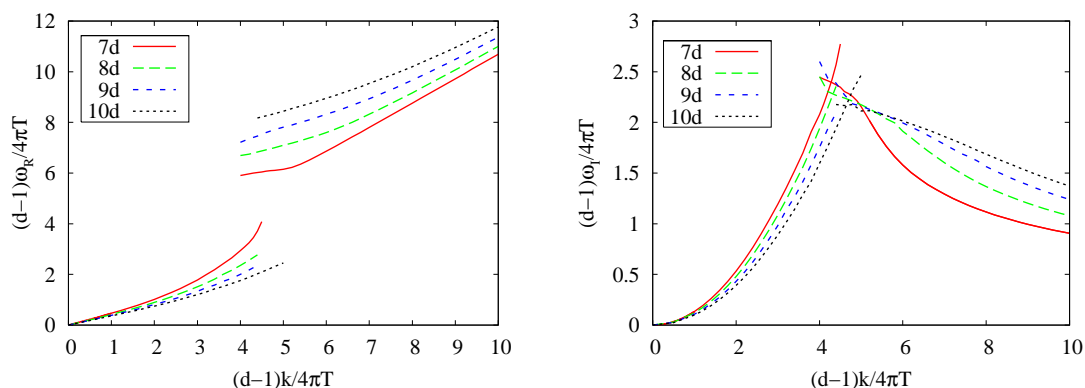
**Figure 7.** Dispersion relations for the dominant scalar QNM (the hydrodynamic mode) of the plane-symmetric AdS black hole in  $d = 4, 5, 6$  dimensions, as indicated.

in the graphics of figure 7. The fundamental QNM of the scalar perturbations is in fact the hydrodynamic (sound-wave) mode. For large wavenumber values, the behavior of the real and imaginary parts of the frequency is similar to that of the tensorial and vectorial sectors, but it is quite different for small values of  $q$ . Such a difference can be attributed to fact that the dominant scalar QNM in the regime of small wavenumbers is a hydrodynamic mode with nonvanishing real part.

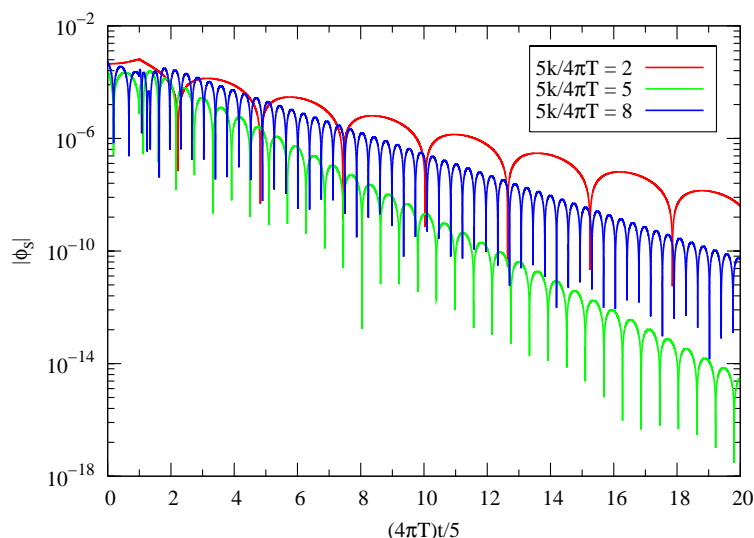
Figure 8 displays the numerical results for the QN frequencies of scalar gravitational perturbations for  $d = 7, \dots, 10$  spacetime dimensions. In such cases, the dispersion relations were obtained by the time-domain evolution method. This method in general gives us information only on the dominant (lowest- $\omega_I$ ) mode, but as apparent in figure 8 we were able to obtain the dispersion relations of the fundamental and the first excited modes in the region  $4 \lesssim q \lesssim 5$ . The results shown in such figure indicate the existence of a critical wavenumber value from which on the first non-hydrodynamic QNM is the dominant mode of the scalar perturbations. Such a result appears in the form of a gap in the real part of the frequencies (left panel in figure 8) and as a crossing of two curves  $\omega_I(q)$  in the right panel of figure 8. This behavior of the scalar QNM in higher-dimensional spacetimes ( $d > 6$ ) is completely different from what happens in  $d = 4, 5$  and 6 dimensions, where the hydrodynamic scalar QNM dominates all of the spectrum. A similar behavior with two concurrent dominant modes in some region of the spectrum was also found for the scalar-type gravitational perturbations of an asymptotically flat black string in  $d = 5, 6$  and 7 dimensions [86]. However, in contrast with the black string case of ref. [86], we do not find purely damped modes in this sector of the gravitational perturbations.

A few samples of the time-domain evolution profile of the scalar perturbations are shown in figure 9. The oscillatory decay, which is characteristic of a QNM with  $\omega_R \neq 0$ , dominates the intermediate- and late-time behavior of the wave function  $\Phi_S$ . Again we do not see any power-law tail at late stages.

The dispersion relation for the first five non-hydrodynamic scalar QNM in six-dimensional spacetime have a behavior very similar to the hydrodynamic mode and are shown in figure 10. The sound-wave mode is also plotted in these graphs (the dashed line

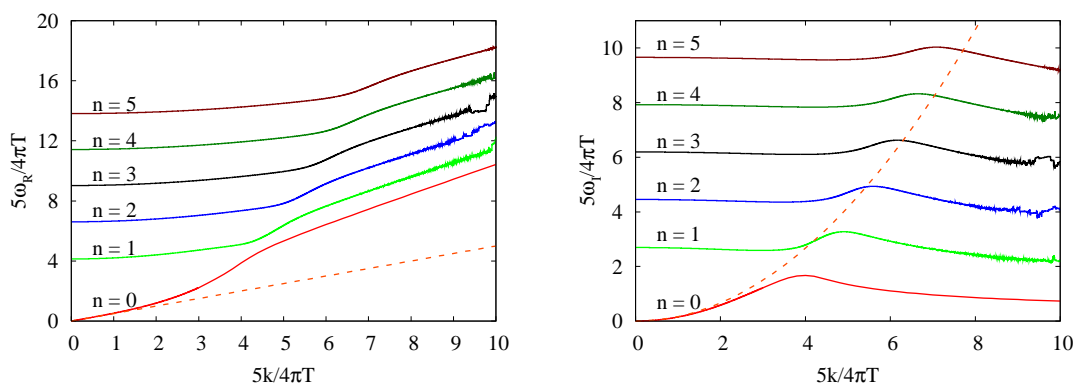


**Figure 8.** Dispersion relations for the dominant scalar QNM in  $d = 7, 8, 9$  and  $10$  dimensions, as indicated. Exceptionally in the region  $4 \lesssim \mathfrak{q} \lesssim 5$ , we present both the fundamental and the first excited modes obtained from the time-evolution method. Notice that the hydrodynamic QNM dominates in the regime of small  $\mathfrak{q}$ , while the first non-hydrodynamic mode is the dominant mode in the high- $\mathfrak{q}$  regime.



**Figure 9.** Time-evolution profiles of the scalar mode perturbation for selected values of  $\mathfrak{q}$  in  $d = 6$  (from top to bottom, the curves are for  $\mathfrak{q} = 2, 8, 5$ , respectively).

in each panel). We again can see that the numerical results agree with the analytical results of equation (5.6) (dashed lines) in the limit  $\mathfrak{w}, \mathfrak{q} \ll 1$ . It is also seen that the peak (maximum) in the imaginary part of the frequency is smoothed out as the overtone number  $n$  increases, and the position of the peak is displaced towards higher  $\mathfrak{q}$ -values. Such a behavior is similar to the  $d = 5$  case. However, it is important to note that the peaks for higher overtones ( $n \geq 1$ ) in  $d = 4$  are located at the origin,  $\mathfrak{q} = 0$  (cf. ref. [51]). Aside this fact, it seems that the overall profile of the dispersion relations does not strongly depend on the number of spacetime dimensions, at least for  $d = 4, 5$  and  $6$ . For scalar perturbations, the Horowitz-Hubeny method presented numerical problems also in six dimensions, as seen in the wiggling curves for high overtones and high values of the wavenumber  $\mathfrak{q}$  in figure 10. As



**Figure 10.** Dispersion relations for the hydrodynamic mode ( $n = 0$ ) and for the first five scalar non-hydrodynamic QNM in six dimensions. The dashed lines correspond to the sound-wave mode of equation (5.6). The wiggles in the plots are due to numerical errors in the Horowitz-Hubeny method.

far as we could check, the numerical error settles in as  $q$  grows and the convergence of the method becomes very sensitive to the numerical precision. Even with these convergence problems, the data in figure 10 is good enough to conclude that, for higher values of  $q$ , the dispersion relations for the scalar QNM of the AdS black branes behave in a similar way as the vectorial and tensorial QNM studied above (cf. figures 2 and 6).

### 6.5 More comments on the numerical results

The QNM spectra give us also information about the stability of the black hole background spacetime. Following this approach, Kodama and Ishibashi [60] have proved that black branes are stable against tensorial and vectorial perturbations in all dimensions, and against scalar perturbations in  $d = 4$ . The stability of higher-dimensional black branes against scalar perturbations from the analytical point of view is still an open problem. Our results for the tensorial and vectorial sectors are consistent with the results by Kodama and Ishibashi [60] and for the scalar sector we have not found unstable modes in  $d = 4$ , as expected. Moreover, we have not found frequencies with negative imaginary parts for small and intermediary wavenumbers  $q$  in  $d \geq 5$  dimensions. In order to investigate a possible instability for large wavenumbers we can consider the asymptotic behavior of the dispersion relations. In [62] it was observed that the analytical prediction by Festuccia and Liu [61] describes quantitatively the numerical results if multiplied by a real function which depends only on the overtone  $n$ . Such results show that the imaginary parts of the QN frequencies are positive for all dimensions, which strongly suggests that these asymptotically AdS black branes are stable against general gravitational perturbations.

In connection with the stability problem, a word of caution should be given here about the geometric interpretation of the background spacetime considered in this work. The plane-symmetric asymptotically AdS black holes (black branes) should not be confused with other higher-dimensional extended black objects appearing in the literature. In particular, as we have shown above, the black-brane spacetime (2.1) does not present any kind of gravitational instability, independent of the parameter values. This is in contrast with

$d$	Tensorial		Vectorial		Scalar	
	$\omega_R$	$\omega_I$	$\omega_R$	$\omega_I$	$\omega_R$	$\omega_I$
4	—	—	1.84942	2.66385	1.84942	2.66385
5	3.11945	2.74668	3.11945	2.74667	3.11945	2.74668
6	4.13591	2.69339	4.13591	2.69339	4.13591	2.69339
7	5.00747	2.61247	5.00760	2.61266	5.00758	2.61249

**Table 4.** The frequencies of the first non-hydrodynamic QNM for all perturbation types, calculated with  $q = 0$ .

other extended objects like the AdS black strings studied in ref. [87] which can present Gregory-Laflamme gravitational instabilities [88] depending of the relation between the longitudinal size of the horizon and the AdS radius.

At this point we are able to confirm the agreement of our numerical results with the analysis of the subsection 5.2. We have seen in that section that in the limit of small wavenumbers and large frequencies the non-hydrodynamic QN frequencies are identical for all the perturbation sectors. In order to see that we choose  $q = 0$  and calculate the frequencies of the first non-hydrodynamic mode for each perturbation sector and for  $d = 4, 5, 6, 7$ . The numerical data are listed in table 4, from where we can see the very good agreement between the analytical and the numerical results. Let us repeat here that, since the Horowitz-Hubeny method presents convergence problems for  $d > 6$ , the results for  $d = 7$  in table 4 were obtained only by lowering the precision requirements. This explains the small differences between the QN frequencies of each sector for  $d = 7$  in table 4. Notice, however, that the results for tensorial sector in  $d = 7$  spacetime dimensions are in agreement with the results of ref. [78].

In table 5 some results obtained by both of the numerical methods used in the present work, namely the Horowitz-Hubeny method and the time domain evolution method, are shown for comparison. We have chosen the fundamental mode ( $n = 0$ ) and the number of dimensions where QNM frequencies were found through both of the methods. It is seen that the two methods yield consistent and satisfactory results for tensorial, vectorial and scalar type perturbations.

## 7 QNM and the AdS/CFT correspondence

### 7.1 Thermalization timescale

According to the AdS/CFT duality, perturbing a black hole in the AdS bulk is equivalent to perturbing a CFT thermal state in the AdS spacetime boundary, and the time evolution of the black hole perturbation describes the time evolution of fluctuations of the thermal state. In particular, the characteristic damping time of a quasinormal mode,  $\tau = 1/\omega_I = (d - 1)/(4\pi T\omega_I)$ , is related to the thermalization timescale of the dual system, i.e., the characteristic time the perturbed thermal system spends to return to thermal equilibrium. This timescale is dominated by the quasinormal mode with lowest imaginary frequency.

Type	Power series				Time evolution	
	$d$	$q$	$\omega_R$	$\omega_I$	$\omega_R$	$\omega_I$
Tensorial	5	10	11.0586	1.75039	11.0594	1.74773
	6	10	11.4325	1.70036	11.4313	1.69703
	7	5	7.47908	2.12521	7.48168	2.12835
Vectorial	5	2	0	1.19612	0	1.19705
	6	2	0	0.872326	0	0.872788
	7	2	0	0.700830	0	0.701345
Scalar	5	10	10.2220	0.536075	10.2713	0.677816
	6	10	10.4304	0.728055	10.4320	0.730181
	7	2	1.02097	0.520255	1.01691	0.533577

**Table 5.** A comparison between some results obtained through the power series and the time evolution methods for  $n = 0$ .

In this subsection we study the decaying timescale of each sector of perturbations. The most interesting case is the vectorial sector, from which we begin the study.

### 7.1.1 Vectorial sector

Vectorial metric perturbations have two dominant quasinormal modes, depending on the perturbation scale (wavelength): the hydrodynamic mode and the first non-hydrodynamic mode. This happens because of a major difference between the behavior of these modes. For the hydrodynamic mode one has  $\omega_I \rightarrow \infty$  when  $q$  increases to infinity. On the other hand, the non-hydrodynamic mode behaves like  $\omega_I \rightarrow 0$  when  $q$  increases. Then, we can see that from  $q = 0$  on the thermalization timescale is dominated by the hydrodynamic mode, this is true even when this timescale takes on the critical value, i.e., its lowest value. Thereafter, the decaying timescale is dominated by the first non-hydrodynamic mode. Here we study this transition and the values of  $q$  and  $\omega_I$  where it occurs, as well as, the value of the critical thermalization timescale in some dimensions.

In the right panel of figure 4 we have the hydrodynamic modes together with the first non-hydrodynamic quasinormal modes for  $d = 4, 5, \dots, 10$  dimensions. From the plots we can see that the hydrodynamic mode dominates for small wavenumber and the non-hydrodynamic mode dominates for intermediate wavenumber. For  $d = 6$  dimensions this behavior is better observed in figure 6, where one can see the transition from non-oscillatory to oscillatory late-time decay. The explicit values for the critical timescale in some dimensions are listed in table 6. It is seen that the values for  $\tau$  increase smoothly with the number of dimensions  $d$ . The critical value for  $q$  in  $d = 5$  is consistent with the value found in ref. [89].

### 7.1.2 Scalar sector

The relaxation timescale related to the scalar gravitational perturbations in  $d = 4, 5, 6$  is dominated entirely by the hydrodynamic QNM. However, the dispersion relation for the imaginary part of the hydrodynamic frequency has a peak where  $\omega_I$  is maximum, so that

$d$	Vectorial			Scalar			Tensorial		
	$\mathfrak{q}$	$\mathfrak{w}_I$	$\tau(T^{-1})$	$\mathfrak{q}$	$\mathfrak{w}_I$	$\tau(T^{-1})$	$\mathfrak{q}$	$\mathfrak{w}_I$	$\tau(T^{-1})$
4	1.935	2.29518	0.104015	3.213	0.414508	0.575942	—	—	—
5	2.622	2.40746	0.132218	3.360	0.913906	0.348296	0	2.69339	0.115889
6	3.100	2.38173	0.167058	4.081	1.66030	0.239648	0	2.74668	0.147727
7	—	—	—	—	—	—	0	2.61200	0.182797

**Table 6.** The values of  $\mathfrak{q}$  and  $\mathfrak{w}_I$  for which  $\tau$  assumes the minimum value for each sector of the gravitational perturbations.

in this peak we have a critical (minimum) thermalization time (cf. figures 7 and 10). The critical values in this case are shown in table 6. In this sector of gravitational perturbations and for  $d \leq 6$ , the values of  $\tau$  decrease smoothly with  $d$ , and we can see that this behavior is the opposite of that of the vectorial and tensorial sectors.

### 7.1.3 Tensorial sector

Firstly we must remember that the tensorial modes arise only in  $d \geq 5$  spacetime dimensions and that the tensorial sector does not present hydrodynamic modes, then the timescale is dominated by the first non-hydrodynamic quasinormal mode whose behavior we can see in figures 1 and 2. The absolute values of the imaginary frequency decrease with  $\mathfrak{q}$ , so that the critical timescale is in  $\mathfrak{q} = 0$ . In this case the values are listed in table 6. In this sector the values for  $\tau$  increase smoothly with  $d$ , and this behavior is the same as that of the vectorial sector.

## 7.2 Causality in the dual CFT plasma

Recently some studies have used the wave-front velocity instead of the group velocity in order to analyze the causality of signal propagation in connection with the physics of the dual CFT plasma [35, 89]. The wave-front velocity is considered a reliable indicator if one wants to study how fast a signal can be transmitted through a dispersive medium because it limits the speed of propagation of a signal through the medium. The wave-front velocity is defined as the velocity with which the onset of a signal travels [89]:

$$v_F = \lim_{\mathfrak{q} \rightarrow \infty} \frac{\mathfrak{w}}{\mathfrak{q}}. \quad (7.1)$$

For  $v_F$  smaller than the speed of light, causality is preserved (in this work,  $c = 1$ ). It also follows that the hydrodynamic vectorial mode violates causality since from eq. (5.3) it has an infinite limit for the wave-front velocity. However, it was shown here that the vectorial hydrodynamic mode does not dominate in the large wavenumber regime. Therefore one should analyze the first non-hydrodynamic QNM, which is the dominant mode in the limit  $\mathfrak{q} \rightarrow \infty$ , and for this mode it holds the general asymptotic formula  $\mathfrak{w} = \mathfrak{q} + \alpha \mathfrak{q}^{-\beta}$ , where  $\beta = (d - 3)/(d + 1)$  and  $\alpha = \alpha_R + i\alpha_I$  is a complex parameter depending on the number of dimensions  $d$  and in the overtone index  $n$ . Therewith, we found numerically that  $\lim_{\mathfrak{q} \rightarrow \infty} (\mathfrak{w}/\mathfrak{q}) = 1$ . This provides a proof of the causality of signal propagation in the dual CFT plasma for any spacetime dimension.



## 8 Final comments and conclusion

In this work we have studied the complete quasinormal spectra of gravitational perturbations of  $d$ -dimensional AdS black branes. Master equations for gravitational perturbations were derived for the Kovtun-Starinets variables [56] in  $d$  spacetime dimensions, and, for comparison, the fundamental equations with the Regge-Wheeler-Zerilli variables [65] were also explored. Among the relevant general results we can mention the proof given in section 4 that for  $d \geq 5$  dimensions, RWZ and KS variables give the same QNM spectra. In this way we have unified two different points of view for a consistent definition of gravitational quasinormal modes: non-deformation of the boundary metric, associated to RWZ variables, and the identification of QN frequencies with poles of correlators, associated to KS variables.

Furthermore, the use of new gauge-invariant variables in  $d$ -dimensional spacetimes allowed the calculation of the hydrodynamic scalar and vectorial QNM of plane-symmetric AdS black holes in  $d$  spacetime dimensions. The expressions (5.3) and (5.6) are in complete agreement with the CFT predictions, furnishing a non-trivial test to the AdS/CFT conjecture [1]. In addition, the results found here for arbitrary  $d$  reproduce exactly the results found in the literature for  $d = 4$  and  $d = 7$  [15, 16, 51], and for  $d = 5$  [13, 14, 56]. Our analytical and numerical calculations confirm the presence of hydrodynamic (sound-wave) modes in any dimension.

The minimum value for the thermalization timescale was obtained for  $d = 4, 5, 6$  and 7. For the tensorial sector the thermalization time is totally determined by the first non-hydrodynamic mode, while in the scalar sector for  $d = 4, 5, 6$  it is dominated by the hydrodynamic QNM alone. More interestingly, the vectorial sector for any  $d$  and the scalar sector for  $d \geq 7$  have two different dominant QNM, the hydrodynamic mode for lower wavenumber values and the first non-hydrodynamic mode for higher wavenumber values.

Even though the numerical methods presented some convergence problems, it was possible to observe a general behavior of the QNM: the dispersion relations for each sector are very similar for all the dimensions, except for a few particularities specific to some specific mode and dimension. In particular, we can mention the local “knee” appearing in the curve of the real dispersion relation of the four-dimensional vectorial sector, and the local maximum in the imaginary parts of frequency that appears for  $d > 4$  in the scalar sector. Our analysis enables us to infer the absence of tails in the time evolution profiles of gravitational perturbations of non-extreme black branes [78]. However, the presence of tails in extreme AdS black holes (and black branes) spacetimes [84] is a problem which should be investigated in detail in a future work.

It is also worth noticing that, as expected, our numerical results do not show any instability of the black branes against tensor- and vector-type perturbations. Moreover, for scalar-type perturbations in  $d \geq 5$  our results suggest the stability of the AdS black branes, what is an important result since the proof of such a stability is still an open question [60].

Another important result is the confirmation that signal propagation in the dual CFT plasma does not violate causality, independently of the number of dimensions of the AdS



spacetime. Although the wave-front velocity related to the hydrodynamic vectorial mode grows with the wavenumber, the signal propagation in the large wavenumber regime is dominated by the first non-hydrodynamic vectorial mode, which obeys a relation of the form  $\omega = \mathbf{q} + \alpha \mathbf{q}^{-\beta}$ , with constant  $\alpha$  and non-negative constant  $\beta$ , resulting in the wave-front velocity  $v_F = 1$ .

Finally, even though we have used two different numerical methods, a power series and a time evolution methods, we cannot confirm the existence of the highly real modes of Daghigh and Green [63, 64]. The existence of such QNM, and the reason why they appear in analytical studies but not in numerical computations, are questions that should be addressed in the future.

## Acknowledgments

This work is partially supported by Fundação para a Ciência e Tecnologia (FCT) - Portugal through project PTDC/FIS/64175/2006. JM thanks Fundação Universidade Federal do ABC (UFABC) for a grant. ASM, CM, and VTZ thank Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) - Brazil for grants.

## References

- [1] J.M. Maldacena, *The large- $N$  limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [[hep-th/9711200](#)] [[SPIRES](#)].
- [2] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [[hep-th/9802150](#)] [[SPIRES](#)].
- [3] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett. B* **428** (1998) 105 [[hep-th/9802109](#)] [[SPIRES](#)].
- [4] H. Boschi-Filho and N.R.F. Braga, *Gauge/string duality and hadronic physics*, *Braz. J. Phys.* **37** (2007) 567 [[hep-th/0604091](#)] [[SPIRES](#)].
- [5] D.T. Son and A.O. Starinets, *Viscosity, black holes and quantum field theory*, *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 95 [[arXiv:0704.0240](#)] [[SPIRES](#)].
- [6] S.S. Gubser, *Heavy ion collisions and black hole dynamics*, *Gen. Rel. Grav.* **39** (2007) 1533 [*Int. J. Mod. Phys. D* **17** (2008) 673] [[SPIRES](#)].
- [7] J. Erdmenger, N. Evans, I. Kirsch and E. Threlfall, *Mesons in gauge/gravity duals*, *Eur. Phys. J. A* **35** (2008) 81 [[arXiv:0711.4467](#)] [[SPIRES](#)].
- [8] E. Iancu, *Partons and jets in a strongly-coupled plasma from AdS/CFT*, *Acta Phys. Polon. B* **39** (2008) 3213 [[arXiv:0812.0500](#)] [[SPIRES](#)].
- [9] S.A. Hartnoll, *Lectures on holographic methods for condensed matter physics*, [arXiv:0903.3246](#) [[SPIRES](#)].
- [10] C.P. Herzog, *Lectures on holographic superfluidity and superconductivity*, *J. Phys. A* **42** (2009) 343001 [[arXiv:0904.1975](#)] [[SPIRES](#)].
- [11] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large  $N$  field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [[hep-th/9905111](#)] [[SPIRES](#)].

- [12] G. Policastro, D.T. Son and A.O. Starinets, *Shear viscosity of strongly coupled  $N = 4$  supersymmetric Yang-Mills plasma*, *Phys. Rev. Lett.* **87** (2001) 081601 [[hep-th/0104066](#)] [[SPIRES](#)].
- [13] G. Policastro, D.T. Son and A.O. Starinets, *From AdS/CFT correspondence to hydrodynamics*, *JHEP* **09** (2002) 043 [[hep-th/0205052](#)] [[SPIRES](#)].
- [14] G. Policastro, D.T. Son and A.O. Starinets, *From AdS/CFT correspondence to hydrodynamics. II: sound waves*, *JHEP* **12** (2002) 054 [[hep-th/0210220](#)] [[SPIRES](#)].
- [15] C.P. Herzog, *The hydrodynamics of M-theory*, *JHEP* **12** (2002) 026 [[hep-th/0210126](#)] [[SPIRES](#)].
- [16] C.P. Herzog, *Sound of M theory*, *Phys. Rev. D* **68** (2003) 024013 [[hep-th/0302086](#)] [[SPIRES](#)].
- [17] P. Kovtun, D.T. Son and A.O. Starinets, *Viscosity in strongly interacting quantum field theories from black hole physics*, *Phys. Rev. Lett.* **94** (2005) 111601 [[hep-th/0405231](#)] [[SPIRES](#)].
- [18] L. Fidkowski, V. Hubeny, M. Kleban and S. Shenker, *The black hole singularity in AdS/CFT*, *JHEP* **02** (2004) 014 [[hep-th/0306170](#)] [[SPIRES](#)].
- [19] V.E. Hubeny, X. Liu, M. Rangamani and S. Shenker, *Comments on cosmic censorship in AdS/CFT*, *JHEP* **12** (2004) 067 [[hep-th/0403198](#)] [[SPIRES](#)].
- [20] T. Hertog and G.T. Horowitz, *Towards a big crunch dual*, *JHEP* **07** (2004) 073 [[hep-th/0406134](#)] [[SPIRES](#)].
- [21] T. Hertog and G.T. Horowitz, *Holographic description of AdS cosmologies*, *JHEP* **04** (2005) 005 [[hep-th/0503071](#)] [[SPIRES](#)].
- [22] G. Festuccia and H. Liu, *Excursions beyond the horizon: black hole singularities in Yang-Mills theories. I*, *JHEP* **04** (2006) 044 [[hep-th/0506202](#)] [[SPIRES](#)].
- [23] S.W. Hawking, *Information loss in black holes*, *Phys. Rev. D* **72** (2005) 084013 [[hep-th/0507171](#)] [[SPIRES](#)].
- [24] D.A. Lowe and L. Thorlacius, *Remarks on the black hole information problem*, *Phys. Rev. D* **73** (2006) 104027 [[hep-th/0601059](#)] [[SPIRES](#)].
- [25] V. Cardoso and J.P.S. Lemos, *Quasinormal modes of Schwarzschild-anti-de Sitter black holes: electromagnetic and gravitational perturbations*, *Phys. Rev. D* **64** (2001) 084017 [[gr-qc/0105103](#)] [[SPIRES](#)].
- [26] V. Cardoso and J.P.S. Lemos, *Quasi-normal modes of toroidal, cylindrical and planar black holes in anti-de Sitter spacetimes: scalar, electromagnetic and gravitational perturbations*, *Class. Quant. Grav.* **18** (2001) 5257 [[gr-qc/0107098](#)] [[SPIRES](#)].
- [27] V. Cardoso, R. Konoplya and J.P.S. Lemos, *Quasinormal frequencies of Schwarzschild black holes in anti-de Sitter spacetimes: a complete study of the overtone asymptotic behavior*, *Phys. Rev. D* **68** (2003) 044024 [[gr-qc/0305037](#)] [[SPIRES](#)].
- [28] A.S. Miranda and V.T. Zanchin, *Quasinormal modes of plane-symmetric anti-de Sitter black holes: a complete analysis of the gravitational perturbations*, *Phys. Rev. D* **73** (2006) 064034 [[gr-qc/0510066](#)] [[SPIRES](#)].
- [29] P. Kovtun, D.T. Son and A.O. Starinets, *Holography and hydrodynamics: diffusion on stretched horizons*, *JHEP* **10** (2003) 064 [[hep-th/0309213](#)] [[SPIRES](#)].

- [30] N. Iqbal and H. Liu, *Universality of the hydrodynamic limit in AdS/CFT and the membrane paradigm*, *Phys. Rev. D* **79** (2009) 025023 [[arXiv:0809.3808](#)] [[SPIRES](#)].
- [31] K.S. Thorne, R.H. Price and D.A. Macdonald, *Black holes: the membrane paradigm*, Yale University Press, New Haven U.S.A. (1986) [[SPIRES](#)].
- [32] M.K. Parikh and F. Wilczek, *An action for black hole membranes*, *Phys. Rev. D* **58** (1998) 064011 [[gr-qc/9712077](#)] [[SPIRES](#)].
- [33] M. Fujita, *Non-equilibrium thermodynamics near the horizon and holography*, *JHEP* **10** (2008) 031 [[arXiv:0712.2289](#)] [[SPIRES](#)].
- [34] R. Baier, P. Romatschke, D.T. Son, A.O. Starinets and M.A. Stephanov, *Relativistic viscous hydrodynamics, conformal invariance and holography*, *JHEP* **04** (2008) 100 [[arXiv:0712.2451](#)] [[SPIRES](#)].
- [35] M. Natsuume and T. Okamura, *Causal hydrodynamics of gauge theory plasmas from AdS/CFT duality*, *Phys. Rev. D* **77** (2008) 066014 [*Erratum ibid.* **D 78** (2008) 089902] [[arXiv:0712.2916](#)] [[SPIRES](#)].
- [36] M. Natsuume and T. Okamura, *A note on causal hydrodynamics for M-theory branes*, *Prog. Theor. Phys.* **120** (2008) 1217 [[arXiv:0801.1797](#)] [[SPIRES](#)].
- [37] J. Mas and J. Tarrío, *Hydrodynamics from the Dp-brane*, *JHEP* **05** (2007) 036 [[hep-th/0703093](#)] [[SPIRES](#)].
- [38] J.I. Kapusta and T. Springer, *Shear transport coefficients from gauge/gravity correspondence*, *Phys. Rev. D* **78** (2008) 066017 [[arXiv:0806.4175](#)] [[SPIRES](#)].
- [39] T. Springer, *Second order hydrodynamics for a special class of gravity duals*, *Phys. Rev. D* **79** (2009) 086003 [[arXiv:0902.2566](#)] [[SPIRES](#)].
- [40] S. Bhattacharyya, S. Lahiri, R. Loganayagam and S. Minwalla, *Large rotating AdS black holes from fluid mechanics*, *JHEP* **09** (2008) 054 [[arXiv:0708.1770](#)] [[SPIRES](#)].
- [41] S. Bhattacharyya, S. Minwalla, V.E. Hubeny and M. Rangamani, *Nonlinear fluid dynamics from gravity*, *JHEP* **02** (2008) 045 [[arXiv:0712.2456](#)] [[SPIRES](#)].
- [42] S. Bhattacharyya et al., *Local fluid dynamical entropy from gravity*, *JHEP* **06** (2008) 055 [[arXiv:0803.2526](#)] [[SPIRES](#)].
- [43] S. Bhattacharyya, R. Loganayagam, I. Mandal, S. Minwalla and A. Sharma, *Conformal nonlinear fluid dynamics from gravity in arbitrary dimensions*, *JHEP* **12** (2008) 116 [[arXiv:0809.4272](#)] [[SPIRES](#)].
- [44] S. Bhattacharyya, S. Minwalla and S.R. Wadia, *The incompressible non-relativistic Navier-Stokes equation from gravity*, *JHEP* **08** (2009) 059 [[arXiv:0810.1545](#)] [[SPIRES](#)].
- [45] S. Bhattacharyya et al., *Forced fluid dynamics from gravity*, *JHEP* **02** (2009) 018 [[arXiv:0806.0006](#)] [[SPIRES](#)].
- [46] I. Fouxon and Y. Oz, *CFT hydrodynamics: symmetries, exact solutions and gravity*, *JHEP* **03** (2009) 120 [[arXiv:0812.1266](#)] [[SPIRES](#)].
- [47] C. Eling, I. Fouxon and Y. Oz, *The incompressible Navier-Stokes equations from membrane dynamics*, [arXiv:0905.3638](#) [[SPIRES](#)].
- [48] A. Núñez and A.O. Starinets, *AdS/CFT correspondence, quasinormal modes, and thermal correlators in N = 4 supersymmetric Yang-Mills theory*, *Phys. Rev. D* **67** (2003) 124013 [[hep-th/0302026](#)] [[SPIRES](#)].

- [49] C.P. Herzog, P. Kovtun, S. Sachdev and D.T. Son, *Quantum critical transport, duality and M-theory*, *Phys. Rev. D* **75** (2007) 085020 [[hep-th/0701036](#)] [[SPIRES](#)].
- [50] I. Amado, K. Landsteiner, S. Montero and C. Hoyos, *Residues of correlators in the strongly coupled  $N = 4$  plasma*, *Phys. Rev. D* **77** (2008) 065004 [[arXiv:0710.4458](#)] [[SPIRES](#)].
- [51] A.S. Miranda, J. Morgan and V.T. Zanchin, *Quasinormal modes of plane-symmetric black holes according to the AdS/CFT correspondence*, *JHEP* **11** (2008) 030 [[arXiv:0809.0297](#)] [[SPIRES](#)].
- [52] J.J. Friess, S.S. Gubser, G. Michalogiorgakis and S.S. Pufu, *Expanding plasmas and quasinormal modes of anti-de Sitter black holes*, *JHEP* **04** (2007) 080 [[hep-th/0611005](#)] [[SPIRES](#)].
- [53] G. Michalogiorgakis and S.S. Pufu, *Low-lying gravitational modes in the scalar sector of the global  $AdS_4$  black hole*, *JHEP* **02** (2007) 023 [[hep-th/0612065](#)] [[SPIRES](#)].
- [54] G. Siopsis, *Low frequency quasi-normal modes of AdS black holes*, *JHEP* **05** (2007) 042 [[hep-th/0702079](#)] [[SPIRES](#)].
- [55] J. Alsup and G. Siopsis, *Low-lying quasinormal modes of topological AdS black holes and hydrodynamics*, *Phys. Rev. D* **78** (2008) 086001 [[arXiv:0805.0287](#)] [[SPIRES](#)].
- [56] P.K. Kovtun and A.O. Starinets, *Quasinormal modes and holography*, *Phys. Rev. D* **72** (2005) 086009 [[hep-th/0506184](#)] [[SPIRES](#)].
- [57] D.T. Son and A.O. Starinets, *Minkowski-space correlators in AdS/CFT correspondence: recipe and applications*, *JHEP* **09** (2002) 042 [[hep-th/0205051](#)] [[SPIRES](#)].
- [58] C.P. Herzog and D.T. Son, *Schwinger-Keldysh propagators from AdS/CFT correspondence*, *JHEP* **03** (2003) 046 [[hep-th/0212072](#)] [[SPIRES](#)].
- [59] K. Skenderis and B.C. van Rees, *Real-time gauge/gravity duality*, *Phys. Rev. Lett.* **101** (2008) 081601 [[arXiv:0805.0150](#)] [[SPIRES](#)].
- [60] H. Kodama and A. Ishibashi, *Stability of generalised static black holes in higher dimensions*, [gr-qc/0312012](#) [[SPIRES](#)].
- [61] G. Festuccia and H. Liu, *A Bohr-Sommerfeld quantization formula for quasinormal frequencies of AdS black holes*, [arXiv:0811.1033](#) [[SPIRES](#)].
- [62] J. Morgan, V. Cardoso, A.S. Miranda, C. Molina and V.T. Zanchin, *Quasinormal modes of black holes in anti-de Sitter space: a numerical study of the eikonal limit*, *Phys. Rev. D* **80** (2009) 024024 [[arXiv:0906.0064](#)] [[SPIRES](#)].
- [63] R.G. Daghigh and M.D. Green, *A detailed analytic study of the asymptotic quasinormal modes of Schwarzschild-anti de Sitter black holes*, *Class. Quant. Grav.* **26** (2009) 125017 [[SPIRES](#)].
- [64] R.G. Daghigh, *The highly real quasinormal modes of Schwarzschild-anti de Sitter black holes*, *JHEP* **04** (2009) 045 [[arXiv:0901.2353](#)] [[SPIRES](#)].
- [65] H. Kodama and A. Ishibashi, *A master equation for gravitational perturbations of maximally symmetric black holes in higher dimensions*, *Prog. Theor. Phys.* **110** (2003) 701 [[hep-th/0305147](#)] [[SPIRES](#)].
- [66] H. Kodama, A. Ishibashi and O. Seto, *Brane world cosmology: gauge-invariant formalism for perturbation*, *Phys. Rev. D* **62** (2000) 064022 [[hep-th/0004160](#)] [[SPIRES](#)].

- [67] J.P.S. Lemos, *Two-dimensional black holes and planar general relativity*, *Class. Quant. Grav.* **12** (1995) 1081 [[gr-qc/9407024](#)] [[SPIRES](#)].
- [68] C.-G. Huang and C.-B. Liang, *A torus-like black hole*, *Phys. Lett. A* **201** (1995) 27 [[SPIRES](#)].
- [69] J.P.S. Lemos, *Three dimensional black holes and cylindrical general relativity*, *Phys. Lett. B* **353** (1995) 46 [[gr-qc/9404041](#)] [[SPIRES](#)].
- [70] R.-G. Cai and Y.-Z. Zhang, *Black plane solutions in four-dimensional spacetimes*, *Phys. Rev. D* **54** (1996) 4891 [[gr-qc/9609065](#)] [[SPIRES](#)].
- [71] J.P.S. Lemos and V.T. Zanchin, *Rotating charged black string and three dimensional black holes*, *Phys. Rev. D* **54** (1996) 3840 [[hep-th/9511188](#)] [[SPIRES](#)].
- [72] A.M. Awad, *Higher-dimensional charged rotating solutions in (A)dS spacetimes*, *Class. Quant. Grav.* **20** (2003) 2827 [[hep-th/0209238](#)] [[SPIRES](#)].
- [73] G.T. Horowitz and A. Strominger, *Black strings and p-branes*, *Nucl. Phys. B* **360** (1991) 197 [[SPIRES](#)].
- [74] E. Berti, V. Cardoso and A.O. Starinets, *Quasinormal modes of black holes and black branes*, *Class. Quant. Grav.* **26** (2009) 163001 [[arXiv:0905.2975](#)] [[SPIRES](#)].
- [75] S. Chandrasekhar, *The mathematical theory of black holes*, Oxford University Press, New York U.S.A. (1983).
- [76] A.L. Fetter and J.D. Walecka, *Theoretical mechanics of particles and continua*, McGraw-Hill, New York U.S.A. (1980).
- [77] J. Natário and R. Schiappa, *On the classification of asymptotic quasinormal frequencies for d-dimensional black holes and quantum gravity*, *Adv. Theor. Math. Phys.* **8** (2004) 1001 [[hep-th/0411267](#)] [[SPIRES](#)].
- [78] G.T. Horowitz and V.E. Hubeny, *Quasinormal modes of AdS black holes and the approach to thermal equilibrium*, *Phys. Rev. D* **62** (2000) 024027 [[hep-th/9909056](#)] [[SPIRES](#)].
- [79] B. Wang, C. Molina and E. Abdalla, *Evolution of a massless scalar field in Reissner-Nordström anti-de Sitter spacetimes*, *Phys. Rev. D* **63** (2001) 084001 [[hep-th/0005143](#)] [[SPIRES](#)].
- [80] B. Wang, C.-Y. Lin and C. Molina, *Quasinormal behavior of massless scalar field perturbation in Reissner-Nordström anti-de Sitter spacetimes*, *Phys. Rev. D* **70** (2004) 064025 [[hep-th/0407024](#)] [[SPIRES](#)].
- [81] R.H. Price, *Nonspherical perturbations of relativistic gravitational collapse. I. Scalar and gravitational perturbations*, *Phys. Rev. D* **5** (1972) 2419 [[SPIRES](#)].
- [82] C. Gundlach, R.H. Price and J. Pullin, *Late-time behavior of stellar collapse and explosions. I. Linearized perturbations*, *Phys. Rev. D* **49** (1994) 883 [[gr-qc/9307009](#)] [[SPIRES](#)].
- [83] A.O. Starinets, *Quasinormal modes of near extremal black branes*, *Phys. Rev. D* **66** (2002) 124013 [[hep-th/0207133](#)] [[SPIRES](#)].
- [84] E.S.C. Ching, P.T. Leung, W.M. Suen and K. Young, *Wave propagation in gravitational systems: late time behavior*, *Phys. Rev. D* **52** (1995) 2118 [[gr-qc/9507035](#)] [[SPIRES](#)].
- [85] D. Forster, *Hydrodynamic fluctuations, broken symmetry, and correlation functions*, Benjamin/Cummings, Reading MA (1975).

- [86] R.A. Konoplya, K. Murata, J. Soda and A. Zhidenko, *Looking at the Gregory-Laflamme instability through quasinormal modes*, *Phys. Rev. D* **78** (2008) 084012 [[arXiv:0807.1897](#)] [[SPIRES](#)].
- [87] T. Hirayama and G. Kang, *Stable black strings in anti-de Sitter space*, *Phys. Rev. D* **64** (2001) 064010 [[hep-th/0104213](#)] [[SPIRES](#)].
- [88] R. Gregory and R. Laflamme, *Black strings and p-branes are unstable*, *Phys. Rev. Lett.* **70** (1993) 2837 [[hep-th/9301052](#)] [[SPIRES](#)].
- [89] I. Amado, C. Hoyos, K. Landsteiner and S. Montero, *Hydrodynamics and beyond in the strongly coupled  $N = 4$  plasma*, *JHEP* (2008) 133 [[arXiv:0805.2570](#)] [[SPIRES](#)].